Voltage-dependent coherent drift modes and turbulent transition regimes in small magnetron devices

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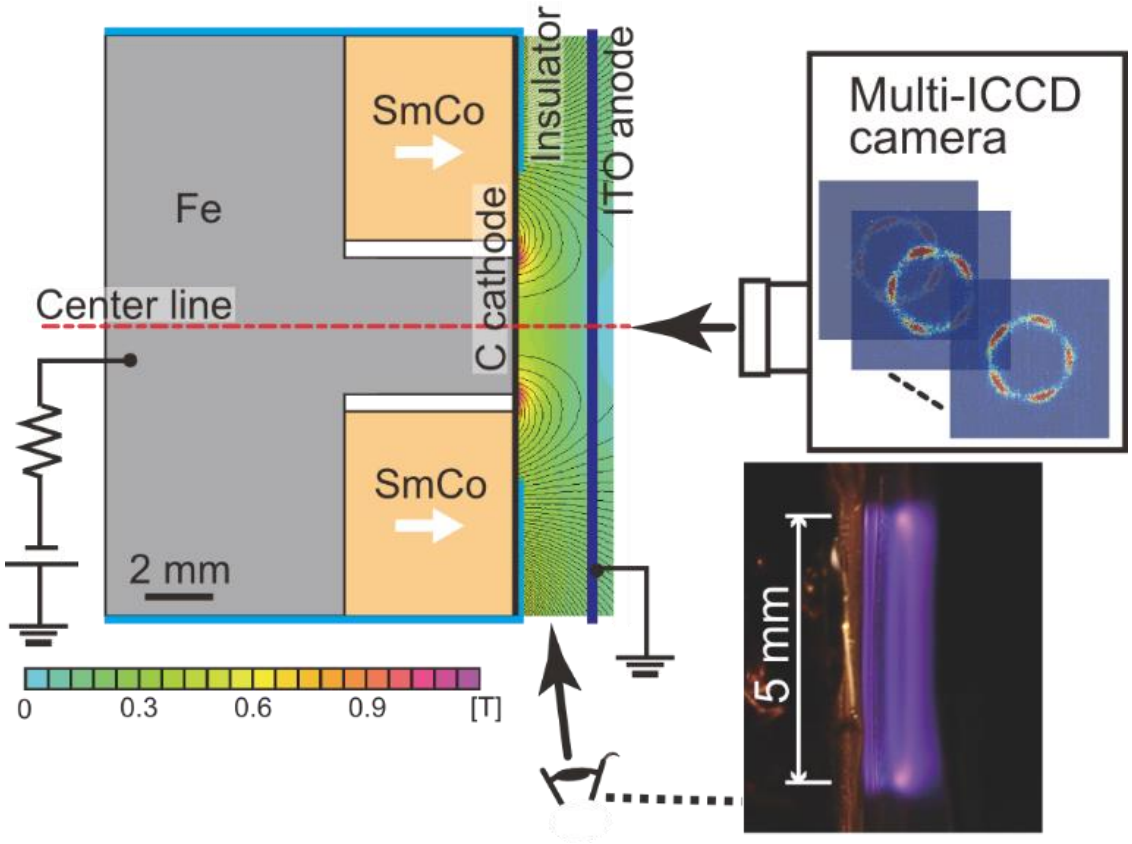
Stanford University

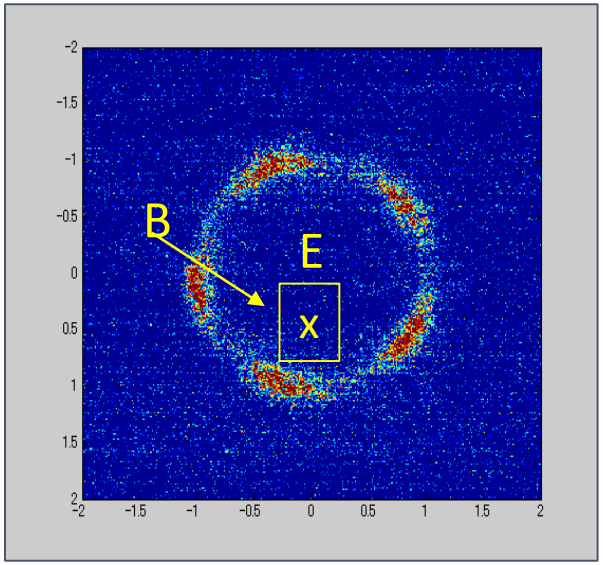
Magnetron Details

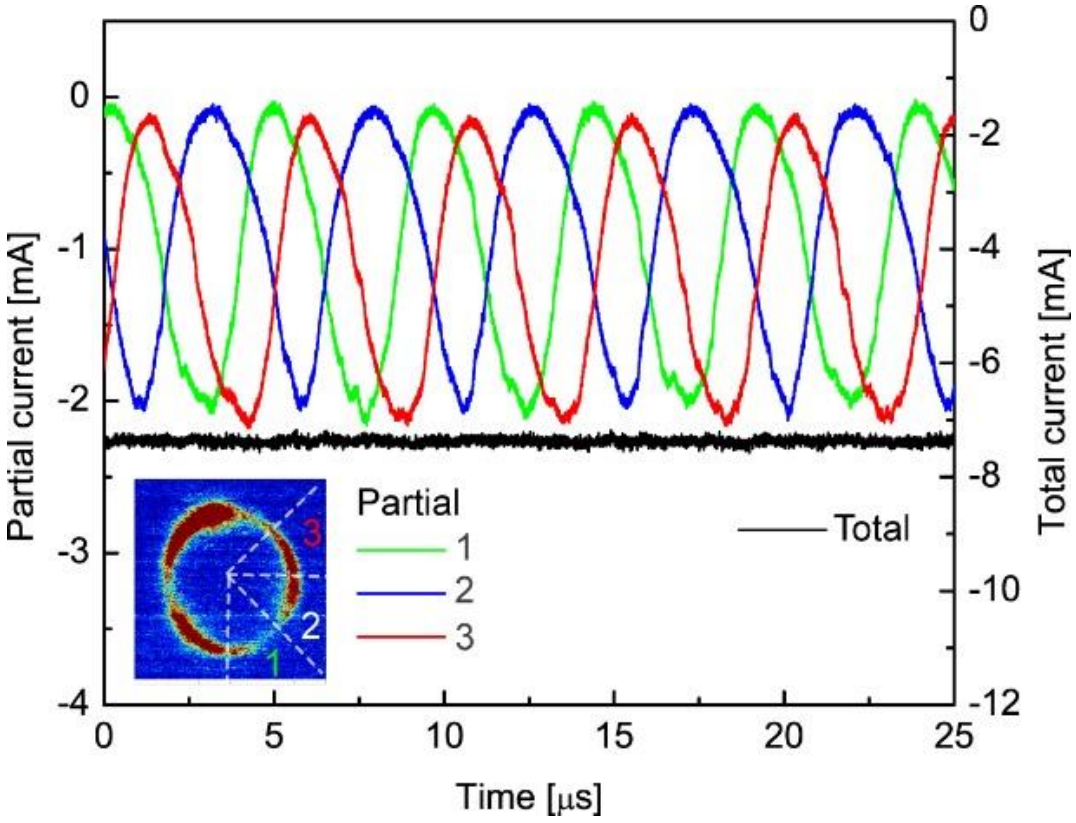


- Very small (~ 5 mm diameter discharge, 2 mm gap)
- Strong radially-inward B (~ 1 T B_{peak}, 0.3 – 5 T B_{plasma})
- Low pressure (~100 mTorr)
- Very strong axial gradients (mm) – expect gradient drifts
- Indium-tin-oxide anode (transparent)

Coherent Rotating Modes

E x B is Counter Clockwise

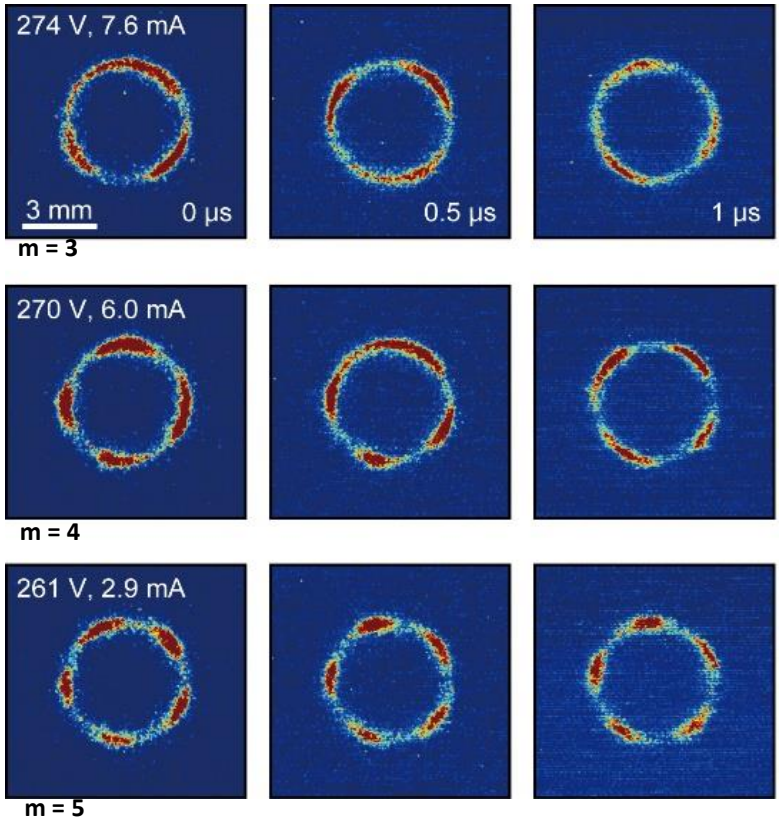


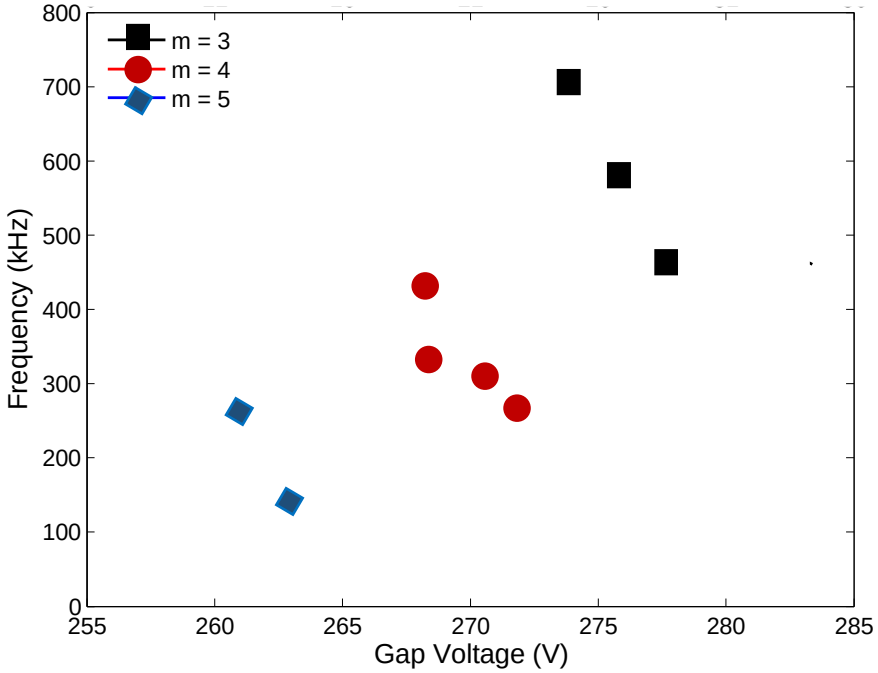


- unambiguous direction confirmed by varying framing rate
- the structures rotate in the - **E x B** direction (retrograde)
- coherent (segmented anode confirms current-carrying)
- the total discharge current shows no evidence of fluctuations

Instability Controlled by Gap Voltage

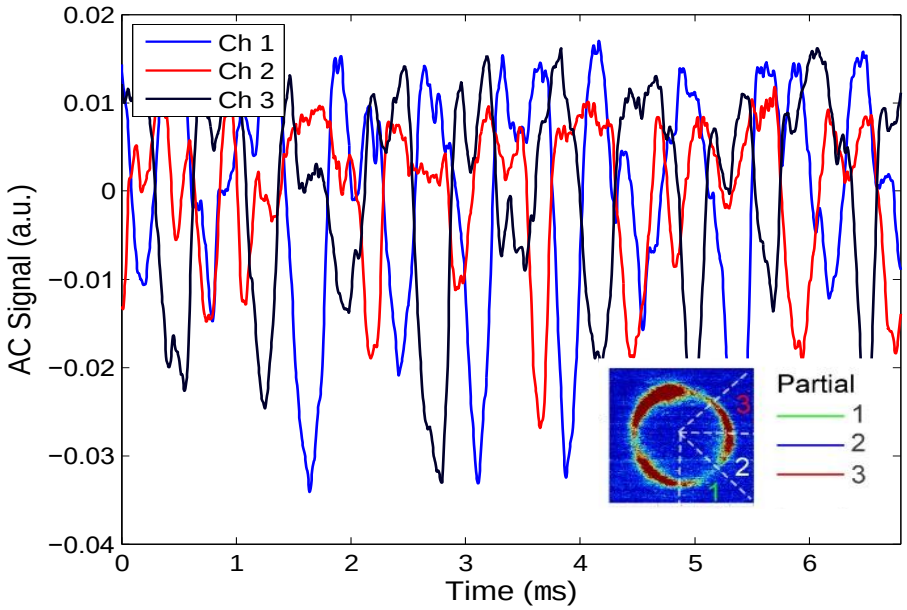
Movie frames

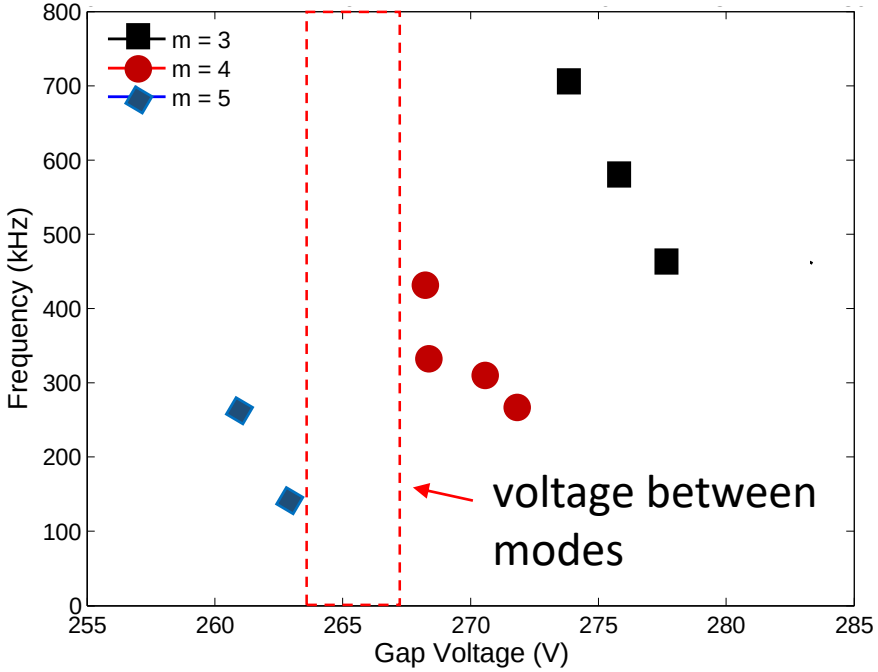




- discharge gap voltage controls structure **frequency** and **mode**
- within mode: frequency **decreases** with increasing voltage (unexpected)!
- wavelength of the modes **increase** (*m* decreases) with increasing voltage

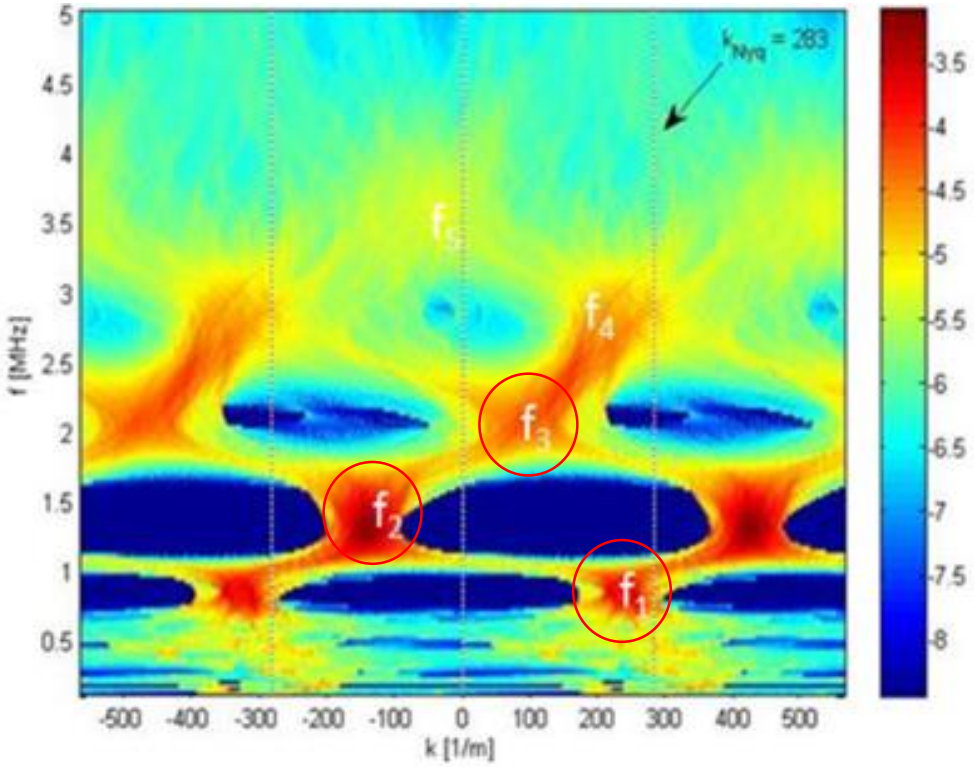
“Turbulent” Regimes between Coherent States



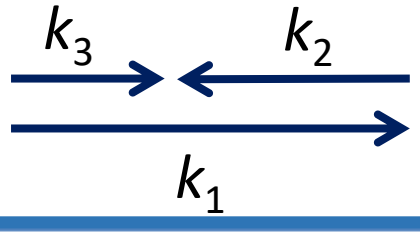


- temporal behavior of oscillations erratic/turbulent between modes
- broad range of frequencies
- anode segments serve as “probes” for wavelet analysis
- wavenumber Nyquist ~ 280 m^{-1}

Three-wave coupling



3-wave mixing in azimuthal waves

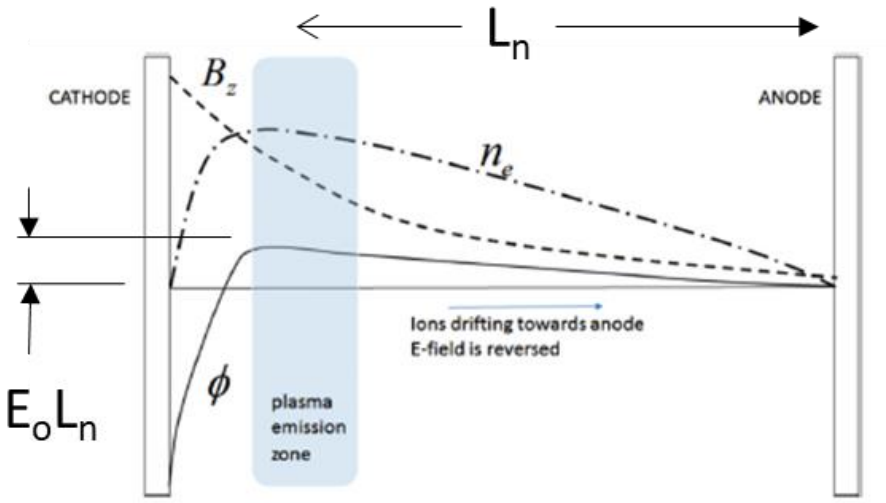


$$f_1(0.85) + f_2(1.31) = f_3(2.16) \quad MHz$$
$$k_1(230) + k_2(-130) = k_3(100) \quad m^{-1}$$

$$f_2(1.31) + f_3(2.16) = f_5(3.47)\ MHz$$
$$k_2(-130) + k_3(100) = k_5(-30)\ m^{-1}$$

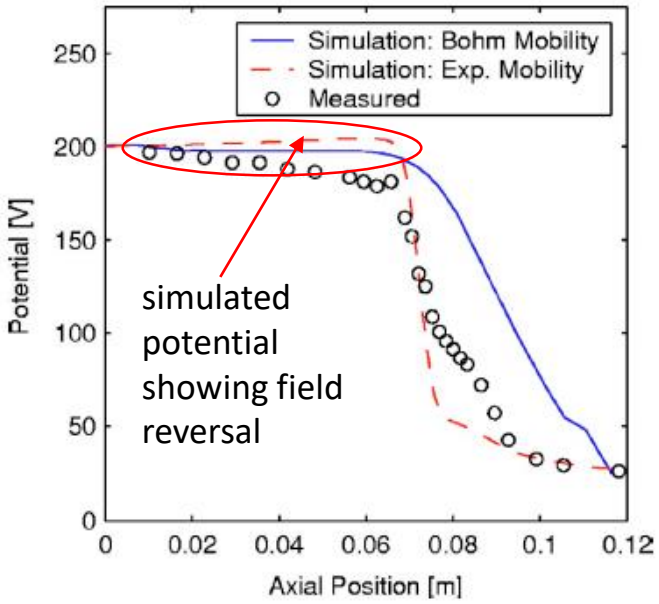
- wavelet analysis reveals high frequency quasi-coherent states (~ five) with strong interconnectivity
- three-wave coupling satisfying momentum and energy selection rules

Hypothesis for Retrograde Motion



- *J* comparable to Hall thrusters (~0.1A/cm^2)
  - similar densities/smaller length scale
- field reversal necessary to restrict diffusion-driven electrons
- plasma rotation in <u>local</u> E x B direction
- field drives ions towards anode

- field reversals predicted in anode region of Hall thrusters
  - potential well ~ 5 V
- causes ions to stream towards anode
- region of strong ionization
- can potentially lead to reversal in ionization (S-H) spoke instabilities



Scharfe, M.K., et al *Physics of Plasmas*, *13*(8), p.083505.

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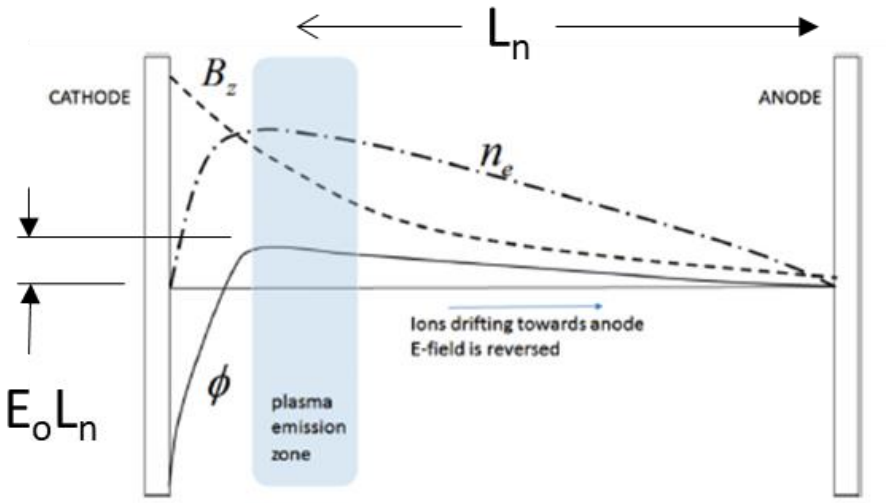

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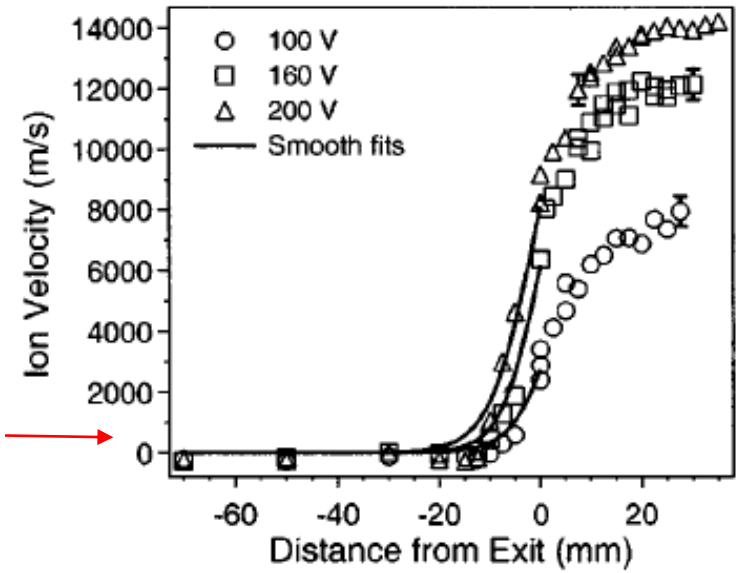
- can potentially lead to reversal in ionization (S-H) spoke instabilities
- anode-streaming ions seen in early LIF data (Meezan et al 2001)



Meezan, et al, 2001. *Physical Review E*, *63*(2), p.026410.

Gradient Drift-wave Theory

Governing Equations

source term : ionization and diffusive loss along B

electron mass and momentum

$$\frac{\partial n}{\partial t} + \boldsymbol{V}_{E\times B}\cdot\nabla n - 2n(\boldsymbol{V}_{E\times B} + \boldsymbol{V}_D)\cdot\nabla lnB_o = v_I n + D_A\frac{\partial n}{\partial z}$$

ion mass

$$\frac{\partial n}{\partial t} + \nabla\cdot n\boldsymbol{v} = v_I n + D_A\frac{\partial n}{\partial z}$$

Ion momentum

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \frac{q}{M}\nabla\phi = 0$$

$$\boldsymbol{V}_{E\times B} = -\frac{\boldsymbol{B}_o}{B_o^2}\times\boldsymbol{E}$$

$$\boldsymbol{V}_D = -\frac{\boldsymbol{B}_o}{B_o^2}\times\frac{kT}{en}\nabla n$$

Linear Perturbation – Fourier Analyzed

$$\frac{\tilde{n}}{n_o} = \frac{k_\perp^2}{(\omega - k_x v_{ox})^2}\frac{e\tilde{\phi}}{M} \qquad \tilde{v}_x = \frac{k_x}{\omega - k_x v_{ox}}\frac{e\tilde{\phi}}{M} \qquad \tilde{v}_y = \frac{k_y}{\omega - k_x v_{ox}}\frac{e\tilde{\phi}}{M}$$ Ions

$$\frac{\tilde{n}}{n_o} = \frac{\omega^* - \omega_D}{\omega - \omega_o - \omega_D + iv^*}\frac{e\tilde{\phi}}{kT} \qquad \tilde{V}_x = -\frac{ik_y}{B_o}\tilde{\phi} = -\frac{ik_y}{B_o}\frac{kT}{en_o}\frac{\omega - \omega_o - \omega_D + iv^*}{\omega^* - \omega_D}\tilde{n}$$ Elec

$$\omega^2 - \left(2k_x v_{ox} + \frac{c_s^2 k_\perp^2}{\omega^* - \omega_o}\right)\omega + (k_x v_{ox})^2 + \frac{c_s^2 k_\perp^2(\omega_o + \omega_D)}{\omega^* - \omega_D} - \frac{ic_s^2 k_\perp^2 v^*}{\omega^* - \omega_D} = 0$$

anode-streaming ion velocity (depends on well depth E_oL_n)

net loss (diffusion less ionization)

$$v^* = k_z^2 D_A - v_I$$

Gradient Drift-wave Theory

Governing Equations

source term : ionization and diffusive loss along B

electron mass and momentum

$$\frac{\partial n}{\partial t} + \boldsymbol{V}_{E\times B}\cdot\nabla n - 2n(\boldsymbol{V}_{E\times B} + \boldsymbol{V}_D)\cdot\nabla lnB_o = \upsilon_I n + D_A\frac{\partial n}{\partial z}$$

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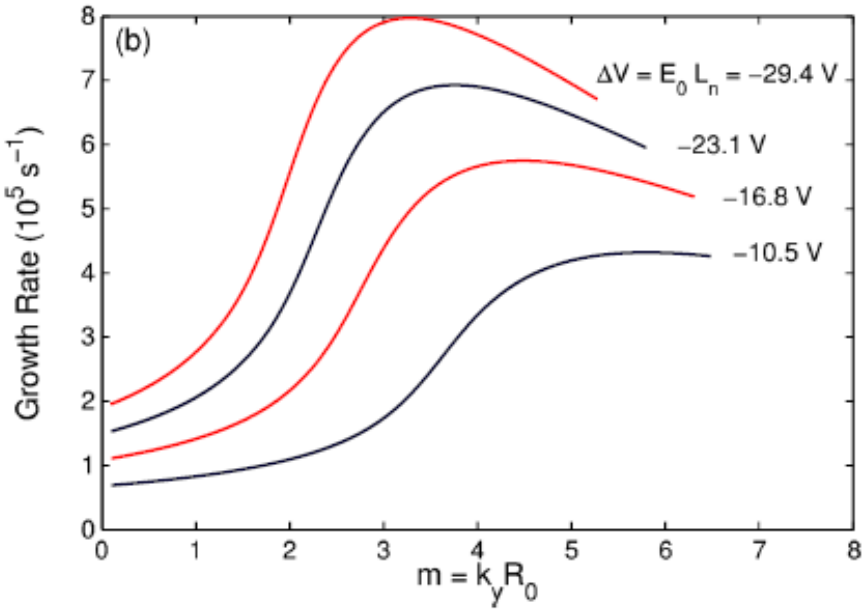
$$\boldsymbol{V}_D = -\frac{\boldsymbol{B}_o}{B_o^2}\times\frac{kT}{en}\nabla n$$

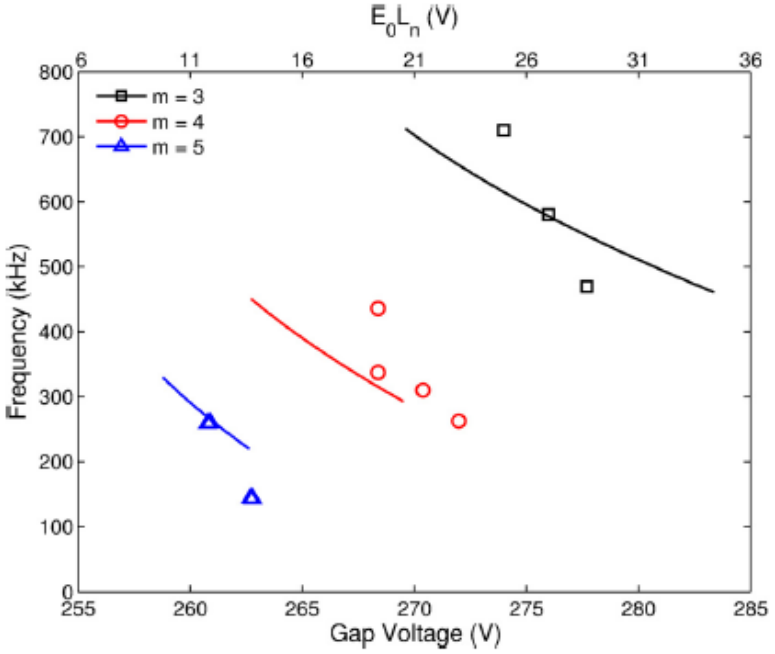
Simon-Hoh (like) Instability but the B-field curvature (drift) term overtakes the density gradient term relieving the requirement of the usual S-H condition that $E_o\frac{dn}{dx} > 0.$

Comparison to Experiments

Growth Rate



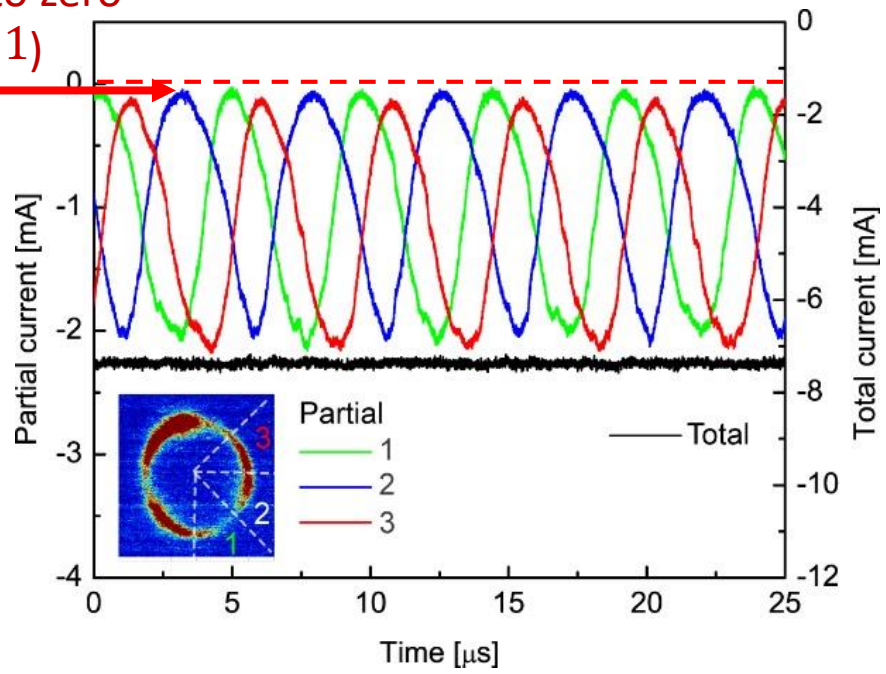
Frequency



- peak growth rate depends on well depth (voltage)
  - increased field (voltage) favors lower mode number (as seen in experiments)
  - expect a hysteresis (also seen in experiments)
- within a mode, increasing voltage decreases frequency

Coherent Fluctuations Drive Transport

Current falls to zero
(so $\tilde{n}/n_o \approx 1$)



Electron Current Density

$$J_e = Re\{e\tilde{n}\tilde{V}_x\} =$$

$$- \frac{k_y n_o kT}{B_o}\left(\frac{\tilde{n}}{n_o}\right)^2 \frac{\nu^*}{(\omega^* - \omega_D)}$$

$$\approx \frac{1}{2} e n_o L_\nabla \nu^* \left(\frac{\tilde{n}}{n_o}\right)^2$$

To match the experiment freqencies

$$\nu^* = 3 \times 10^6 \ s^{-1}$$
$$L_\nabla \approx 10^{-3} \ m$$

If we further assume:

$$n_o = 10^{18} \ m^{-3}$$
$$\left(\frac{\tilde{n}}{n_o}\right)^2 \approx 1$$

$$J_e \approx 0.5 \ A/m^2$$

Consistent with experimental estimates of current density

$$J_e = Re\{e\tilde{n}\tilde{V}_x\}$$ Model

$\nu^* = 3 \times 10^6 \ s^{-1}$
$L_{\nabla B} = 2 \times 10^{-3} \ m$

Summary

- small magnetron discharge can generate very coherent plasma oscillations
- fluctuations propagate opposite external E x B direction
  - likely due to presence of field reversal (potential well) driven by strong gradients in plasma density
  - drift-wave theory describes this behavior fairly well
- "turbulence" between modes
  - evidence of three-wave coupling within this turbulence
- transport during the coherent modes consistent with drift theory
  - S-H "like" with strong curvature drift term
  - cross-field current uniquely determined by gradient length scale and ionization

Acknowledgements: AFOSR

Extra Slides

Linear Perturbation – Fourier Analyzed

$$\frac{\tilde{n}}{n_o} = \frac{\omega^* - \omega_D}{\omega - \omega_o - \omega_D + i\nu^*}\frac{e\tilde{\phi}}{kT} \qquad \tilde{v}_x = \frac{k_x}{\omega - k_x v_{ox}}\frac{e\tilde{\phi}}{M}$$

$$\frac{\tilde{n}}{n_o} = \frac{k_\perp^2}{(\omega - k_x v_{ox})^2}\frac{e\tilde{\phi}}{M} \qquad \tilde{v}_y = \frac{k_y}{\omega - k_x v_{ox}}\frac{e\tilde{\phi}}{M}$$

$$\tilde{V}_x = -\frac{ik_y}{B_o}\tilde{\phi} = -\frac{ik_y}{B_o}\frac{kT}{en_o}\frac{\omega - \omega_o - \omega_D + i\nu^*}{\omega^* - \omega_D}\tilde{n} \qquad Re\{e\tilde{n}\tilde{V}_x\} = \frac{k_y}{B_o}\frac{kT}{en_o}\frac{\nu^*}{\omega^* - \omega_D}\left(\frac{\tilde{n}}{n_o}\right)^2$$

$$\omega^2 - \left(2k_x v_{ox} + \frac{c_s^2 k_\perp^2}{\omega^* - \omega_D}\right)\omega + (k_x v_{ox})^2 + \frac{c_s^2 k_\perp^2(\omega_o + \omega_D)}{\omega^* - \omega_D} - \frac{ic_s^2 k_\perp^2 \nu^*}{\omega^* - \omega_D} = 0$$

anode-streaming ion velocity (depends on well depth $E_o L_n$)

net source (ionization and diffusion)

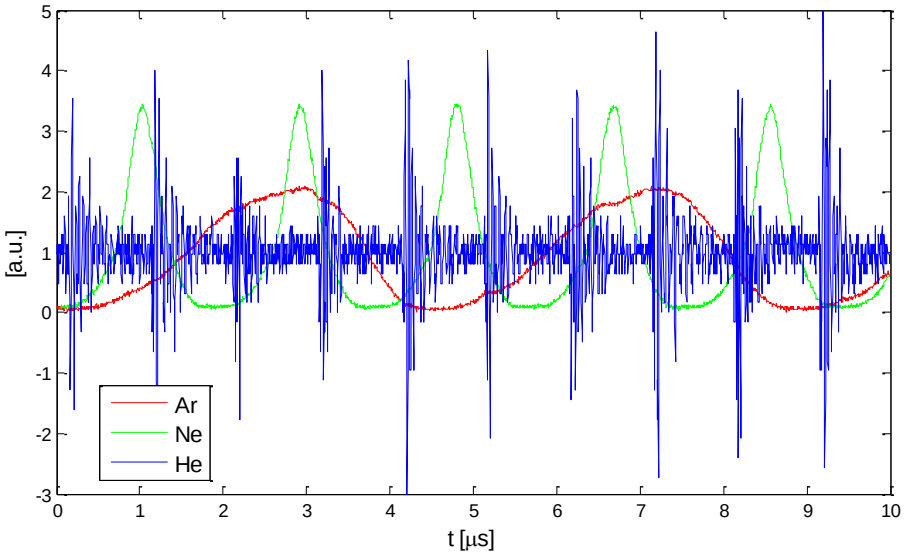
Characteristics Frequencies

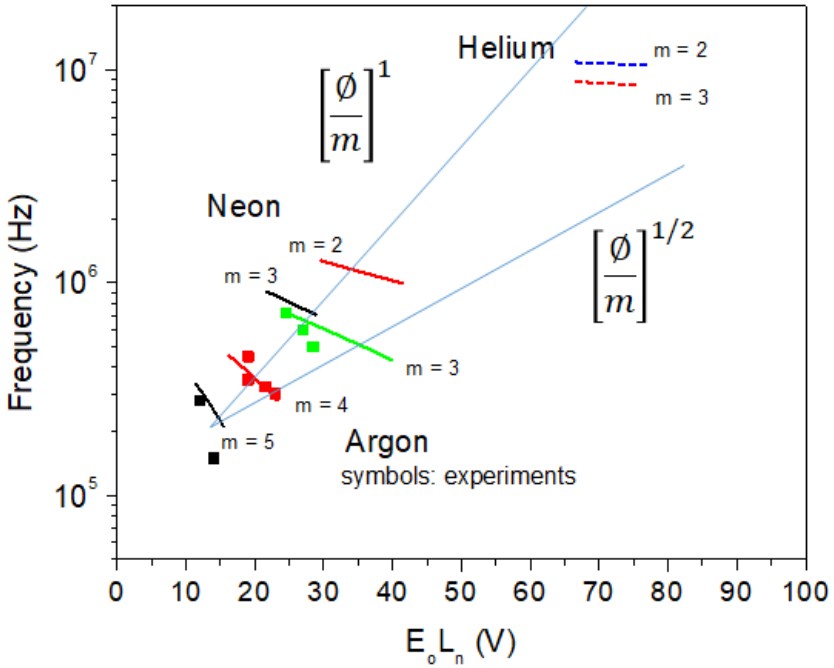
$$\omega^* = -\frac{c_s^2 k_y}{\omega_{ic} L_{\nabla n}} \qquad \omega_D = -\frac{c_s^2 k_y}{\omega_{ic} L_{\nabla B}} \qquad \omega_o = k_y V_{E\times B}$$

Behavior shows strong dependence on ion mass

First experiments with Ne, He

- Frequency/mode increases with decreasing M!
  - Ar (m = 3) =200 kHz,
  - Ne (m = 1) = 500 kHz
  - He (m = 0) = 10 MHz
- Ar (linear), Ne (non-linear), He (pulsating at ~0.5-1MHz)





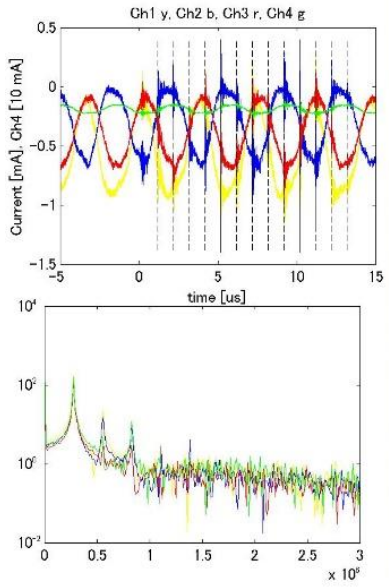
Primary scaling of frequency

- frequency of oscillations scaling:

$$\left[\frac{\emptyset}{M}\right]^{1/2} < f < \left[\frac{\emptyset}{M}\right]^{1}$$

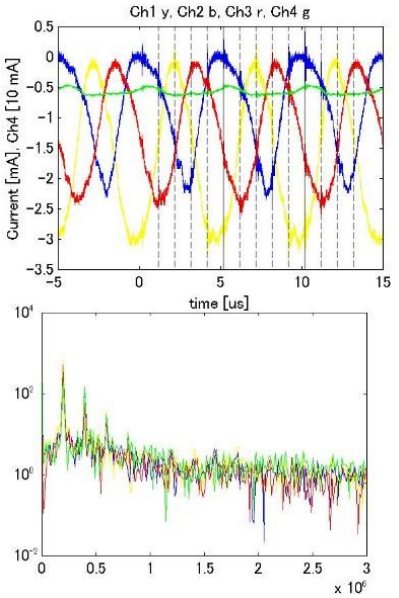
- Primary scaling however does not describe the inverse voltage dependence within modes

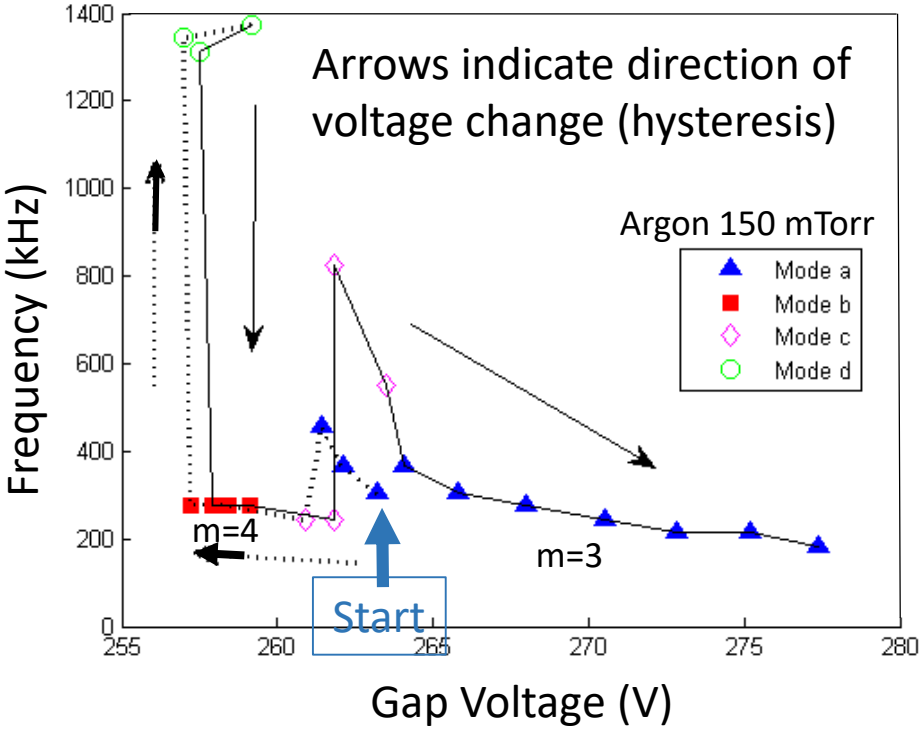
Complex regimes between azimuthal modes

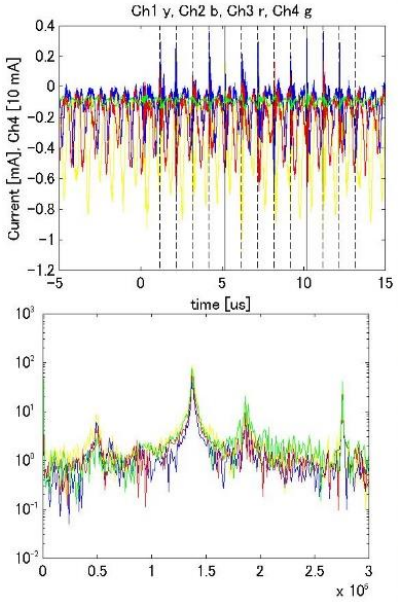


Mode b
coherent
m = 4

Mode a
coherent
m = 3

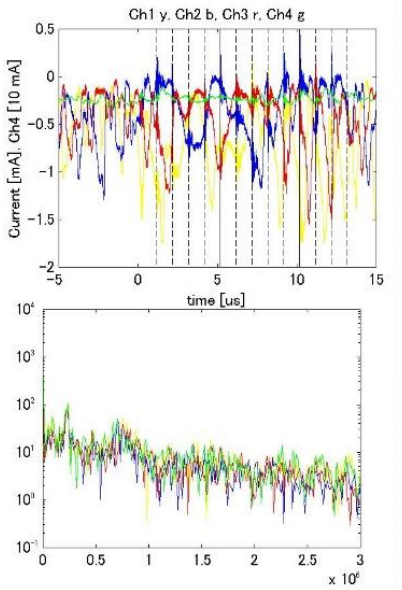




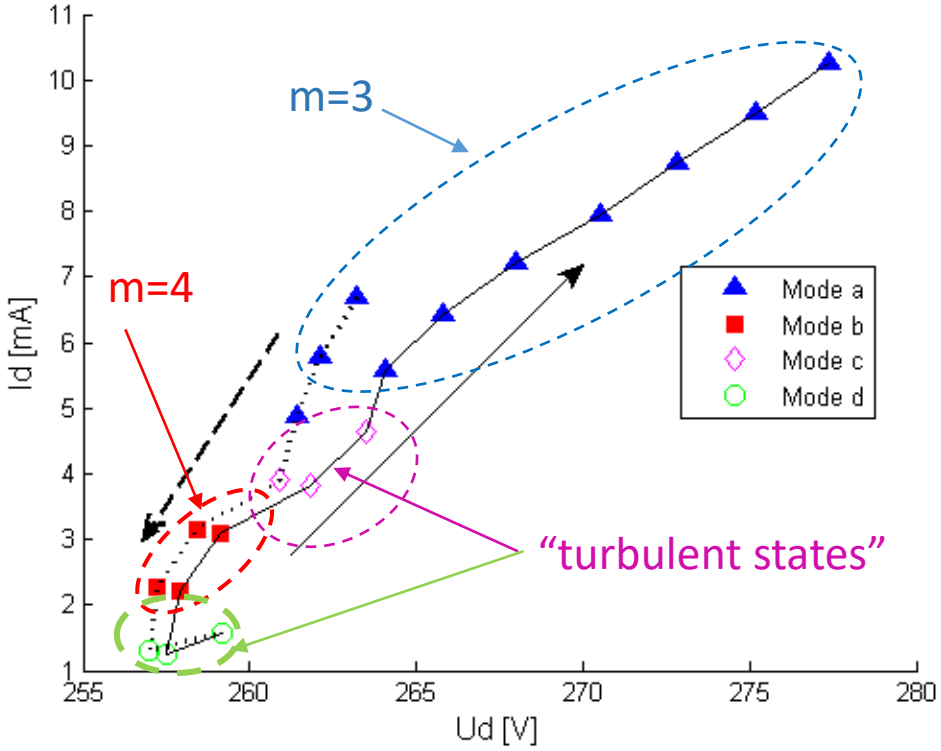


Mode d
Turbulent
(between m = 4/5)

Mode c
Turbulent
(between m = 3/4)



Transport and current flow: resistive behavior



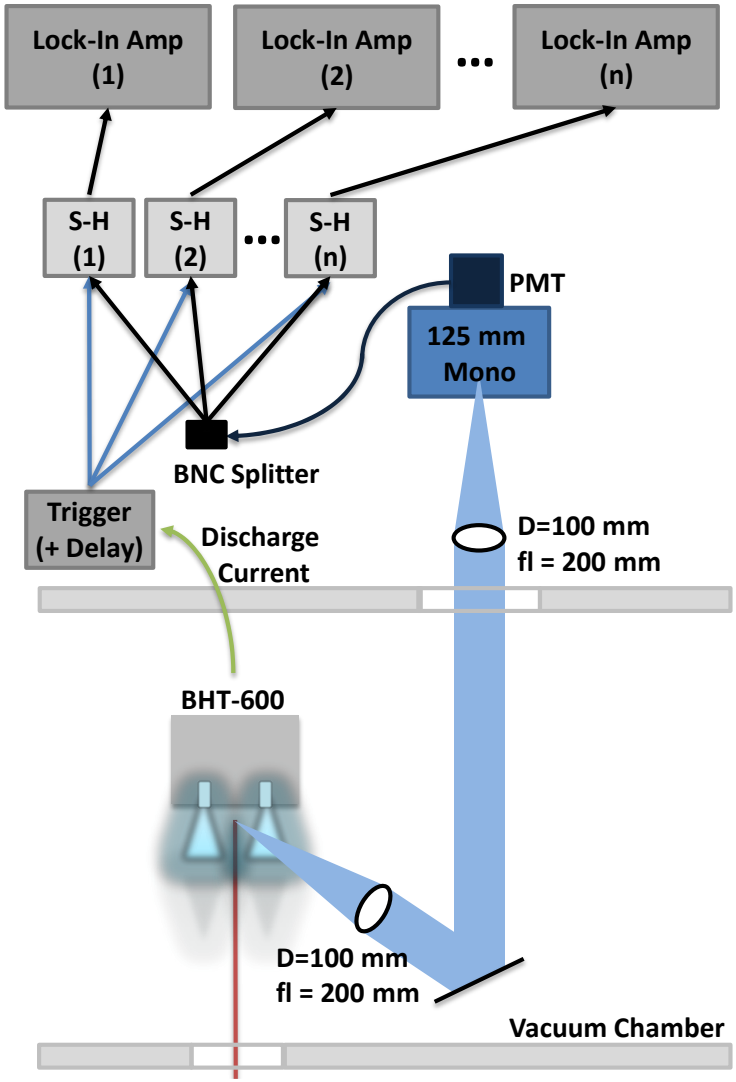
- hysteresis in I-V curve with increasing/decreasing current
- "turbulent state" between m = 3, m = 4 modes (reproducible)
- turbulent states do not greatly enhance the current
- turbulent case near m = 4, m = 5 boundary exhibits non-linear wave-coupling

Breathing mode dynamic via time-resolved ion velocities in a BHT-600 Hall thruster

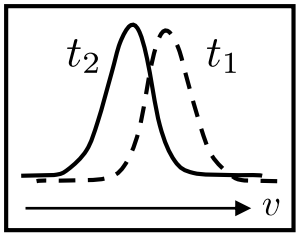
M.A. Cappelli
C.V. Young and A. Lucca-Fabris
W. Hargus, N. McDonald, C. Charles

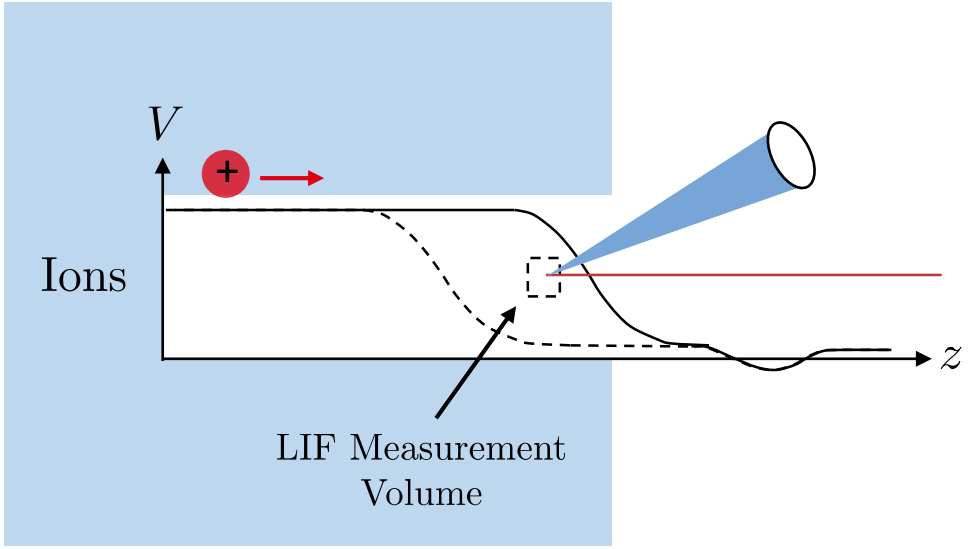
Princeton Workshop 2018

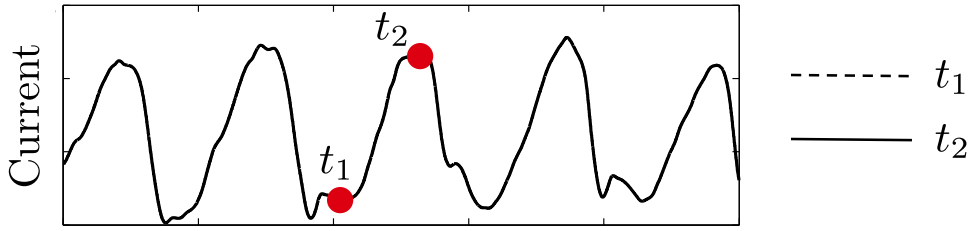
Time-Resolved LIF



Time-Resolved LIF: process only part of PMT signal to obtain one velocity measurement *at one time*







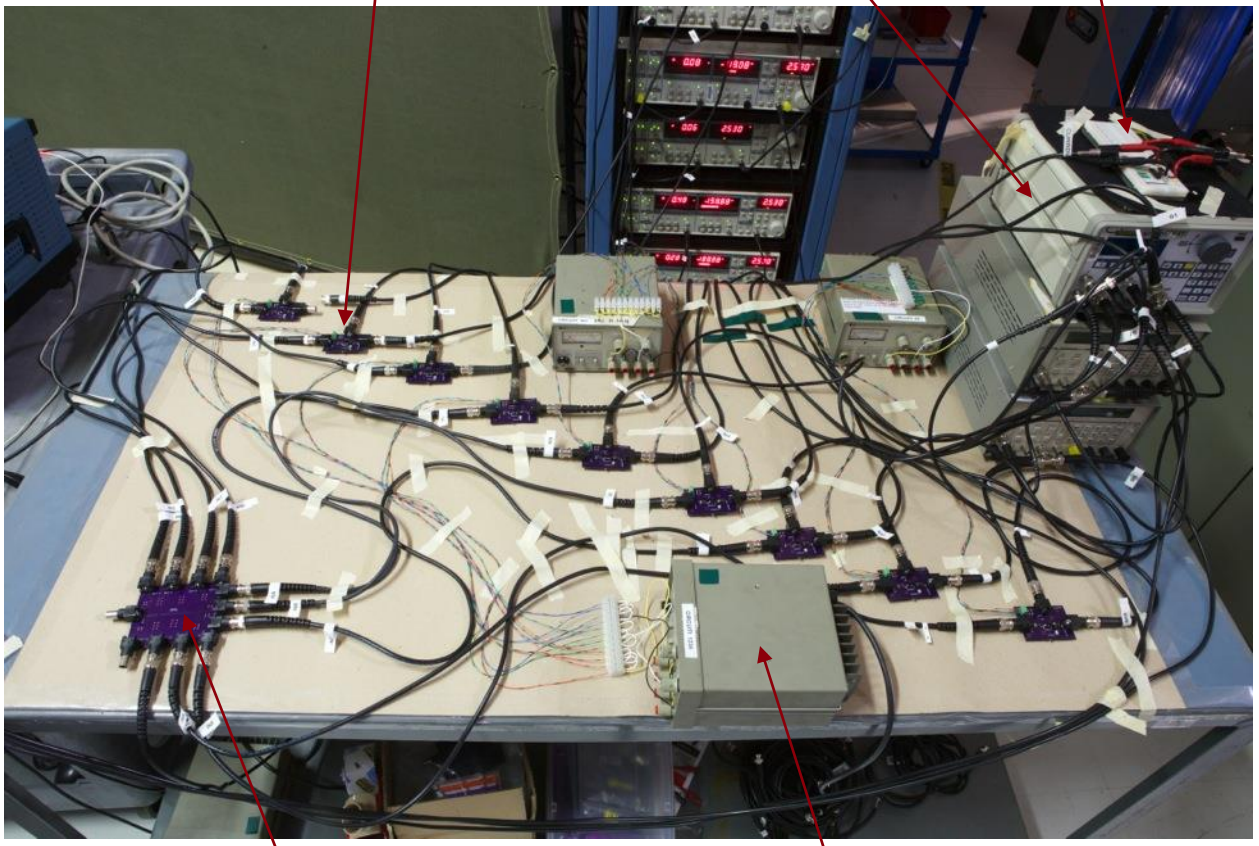
Hardware for Time Resolution

10x SRS Lock-In Amplifiers

9x Sample Hold Circuits

Digital Pulse Delay Generators

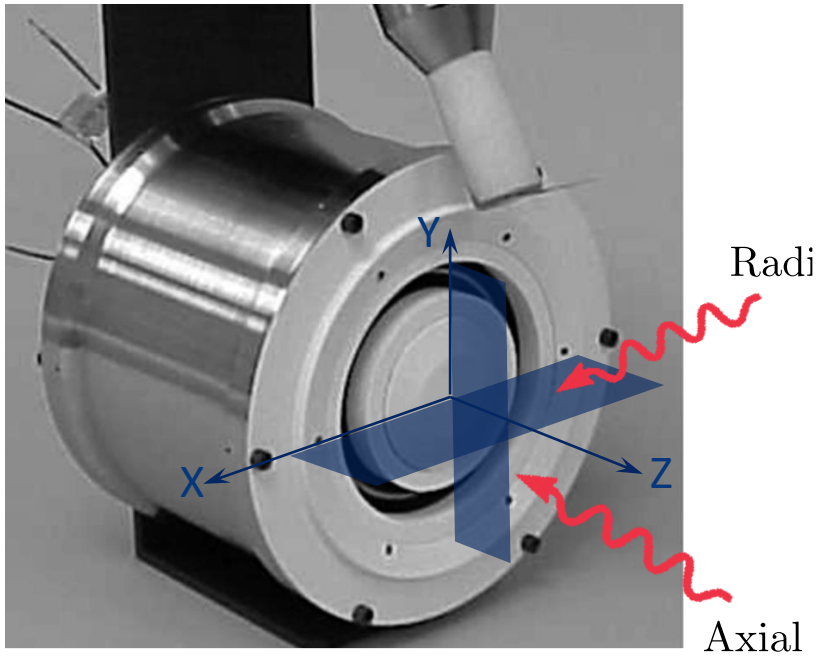
Voltage Comparator

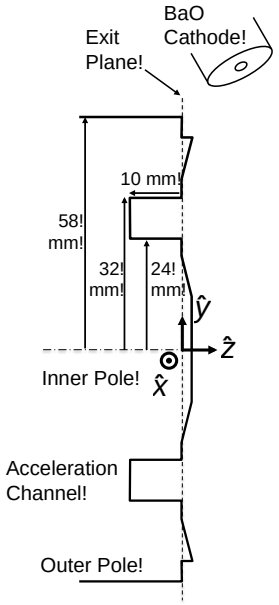


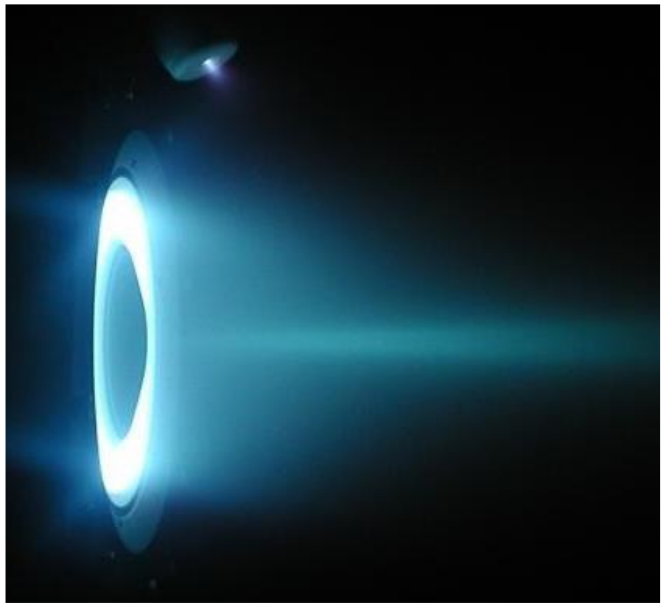
PMT (Voltage) Signal Splitter

Circuit Power Supplies

BHT-600: Time-Resolved Axial + Radial LIF







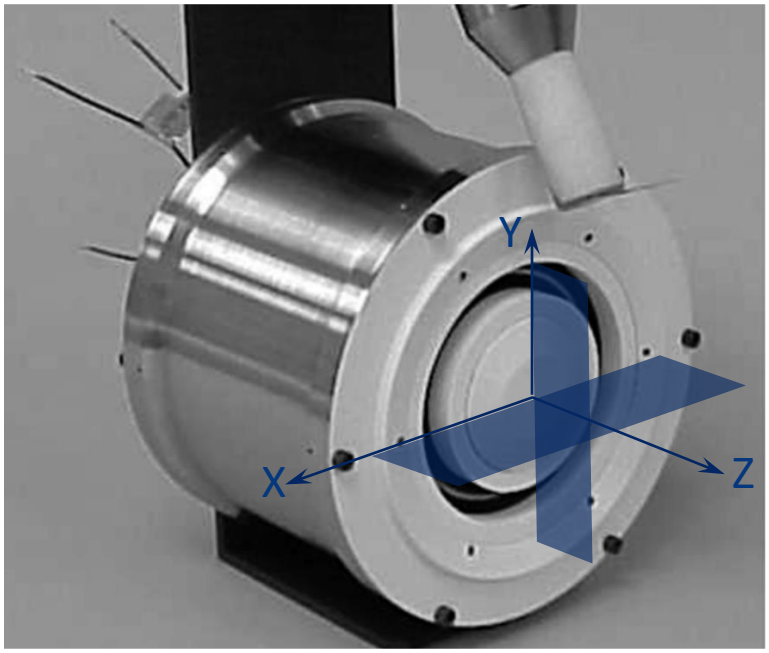
- LIF measures velocity in direction of beam so 2 beams = 2 velocity components (2D)
- First study of 2-D ion dynamics with time-resolved LIF and largest survey of a single operating condition

- By the numbers:

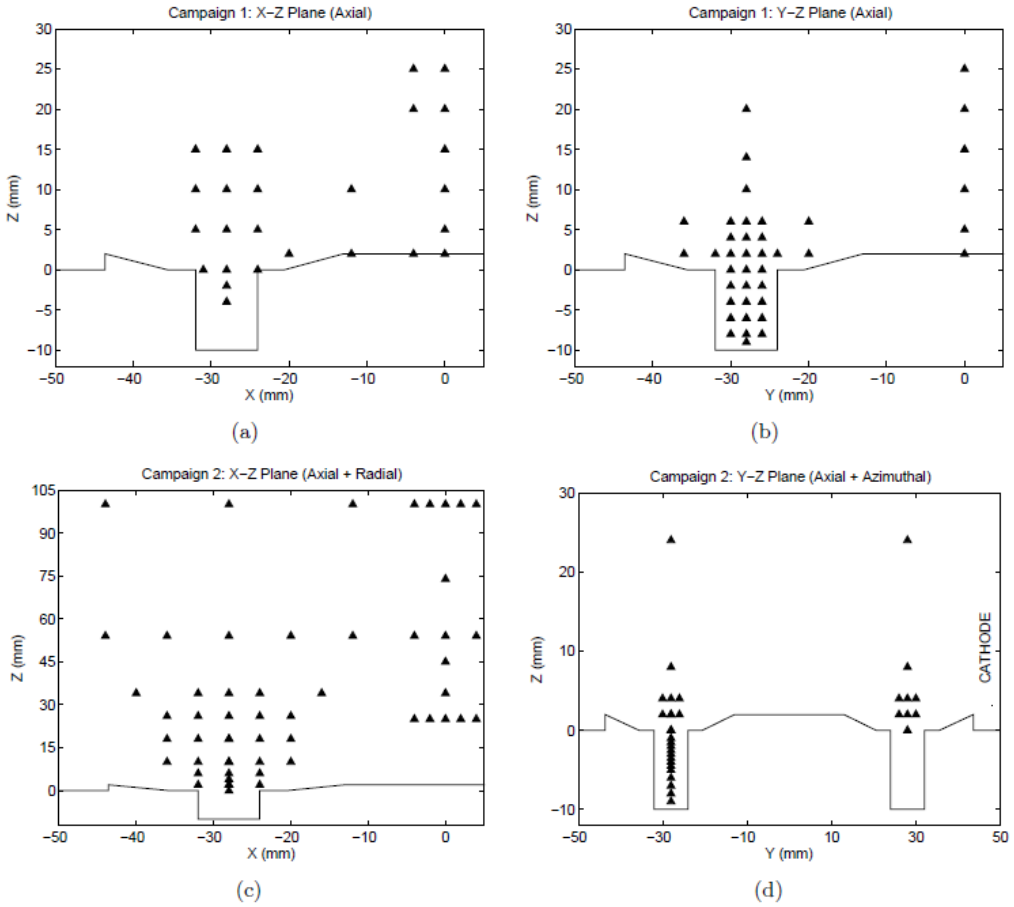
Lock-Ins: 10
Scans/Point: 3
Time points: 28
Time Resolution: 1 μs
Spatial Points: 227
Total Scans: 6356 #VDFs

W. A. Hargus Jr. and C. S. Charles, J. Propul. Power. **26,** 135 (2010)

BHT-600 Hall Thruster



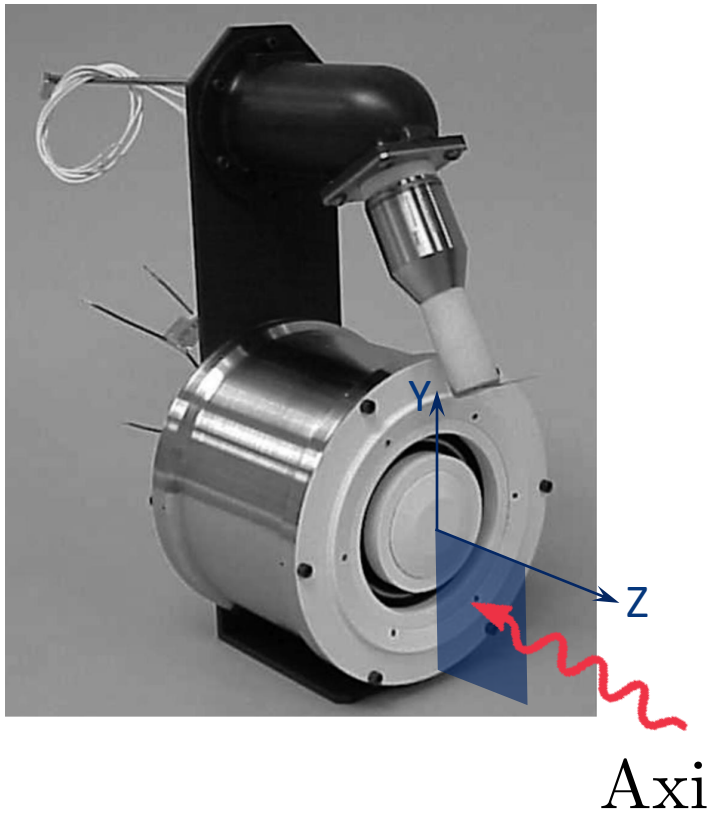
Cathode Bisector



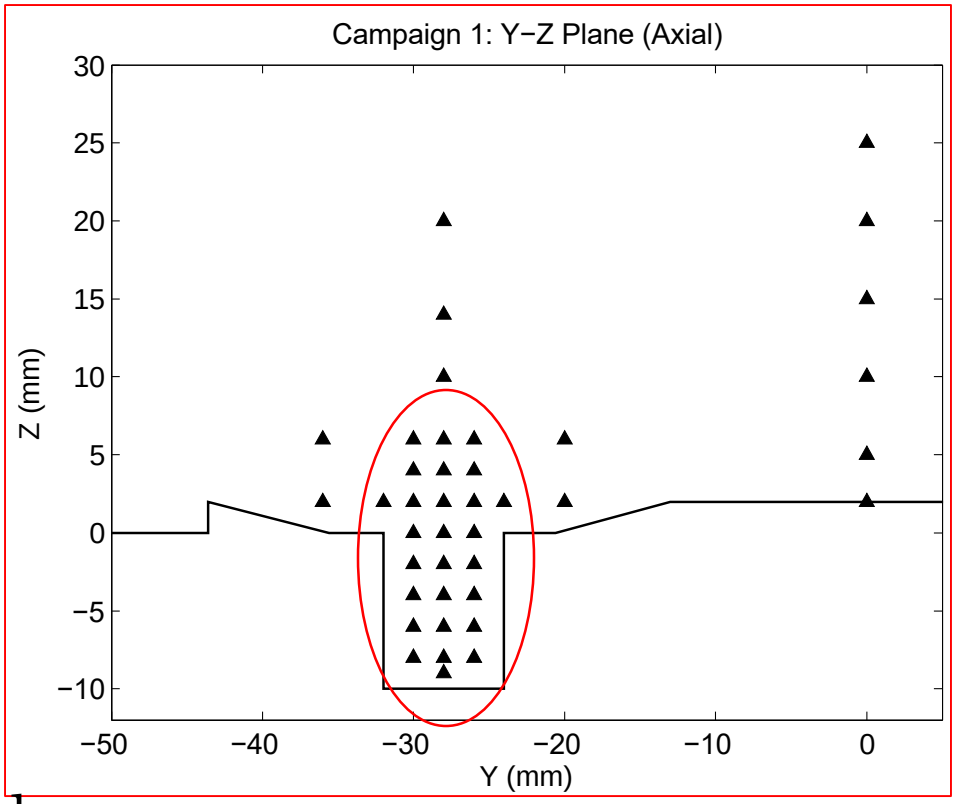
- Time-resolved axial + radial ion LIF in the plume

W. A. Hargus Jr. and C. S. Charles, J. Propul. Power. **26,** 135 (2010)

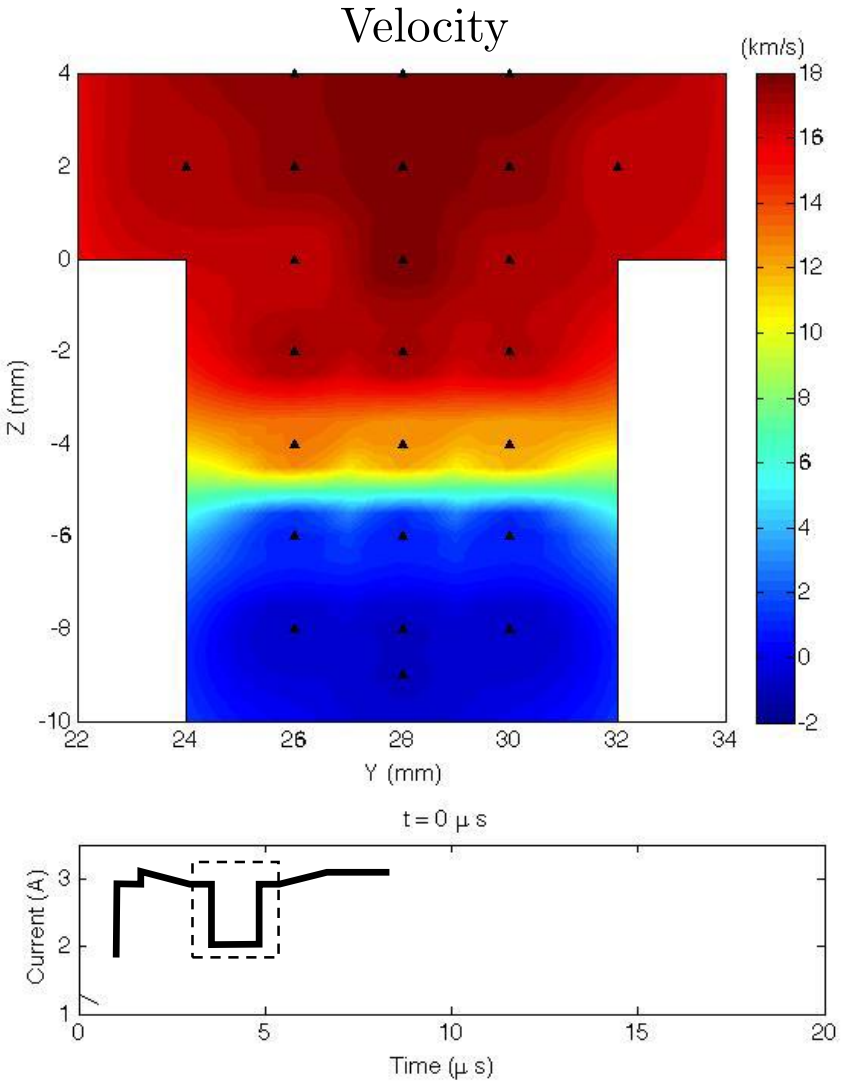
BHT-600 LIF Results: Channel Ion Velocities

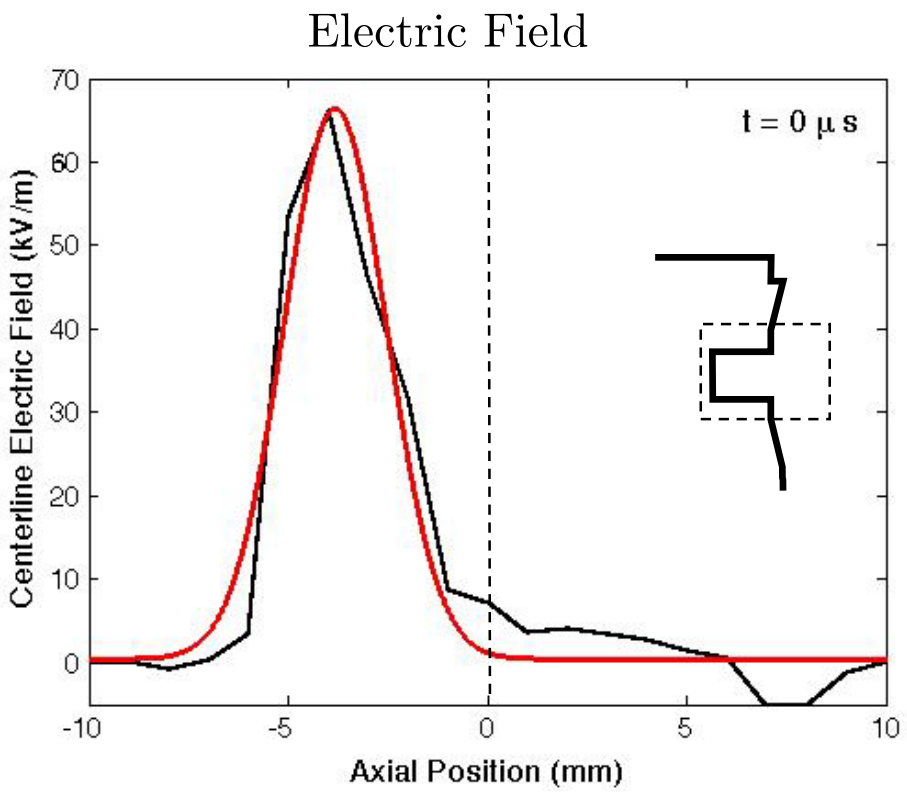


Axial



BHT-600 Results: Time-Resolved Channel

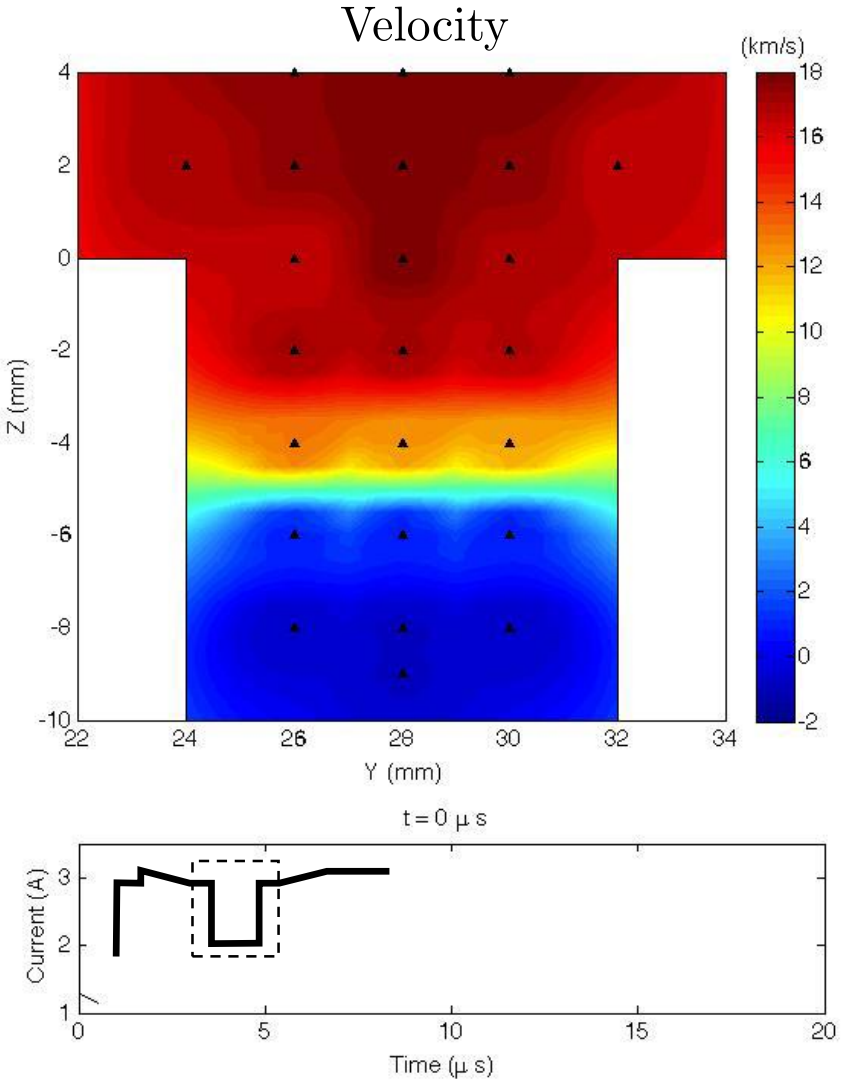




$$E_z = \frac{m_\mathrm{i}}{e}\left(\frac{\partial v_z}{\partial t} + v_z \cdot \frac{\partial v_z}{\partial z}\right)$$

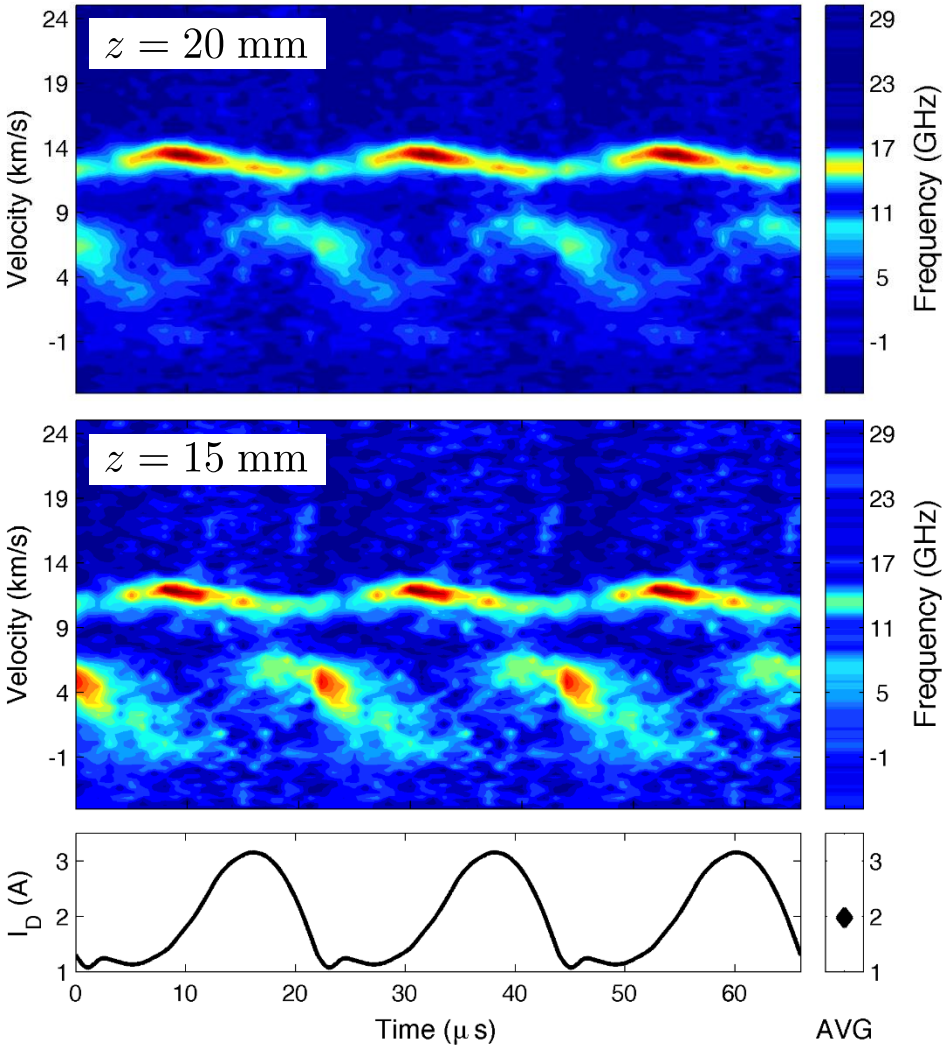
- Moving potential hill captured with LIF

BHT-600 Results: Time-Resolved Channel

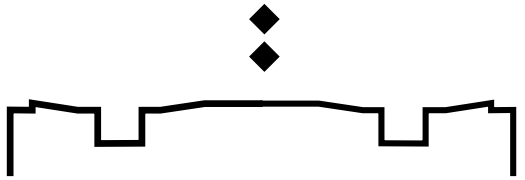


- It seems that the acceleration zone retreats towards the anode when the discharge current decreases

BHT-600 Results: Center Jet, Near Wall



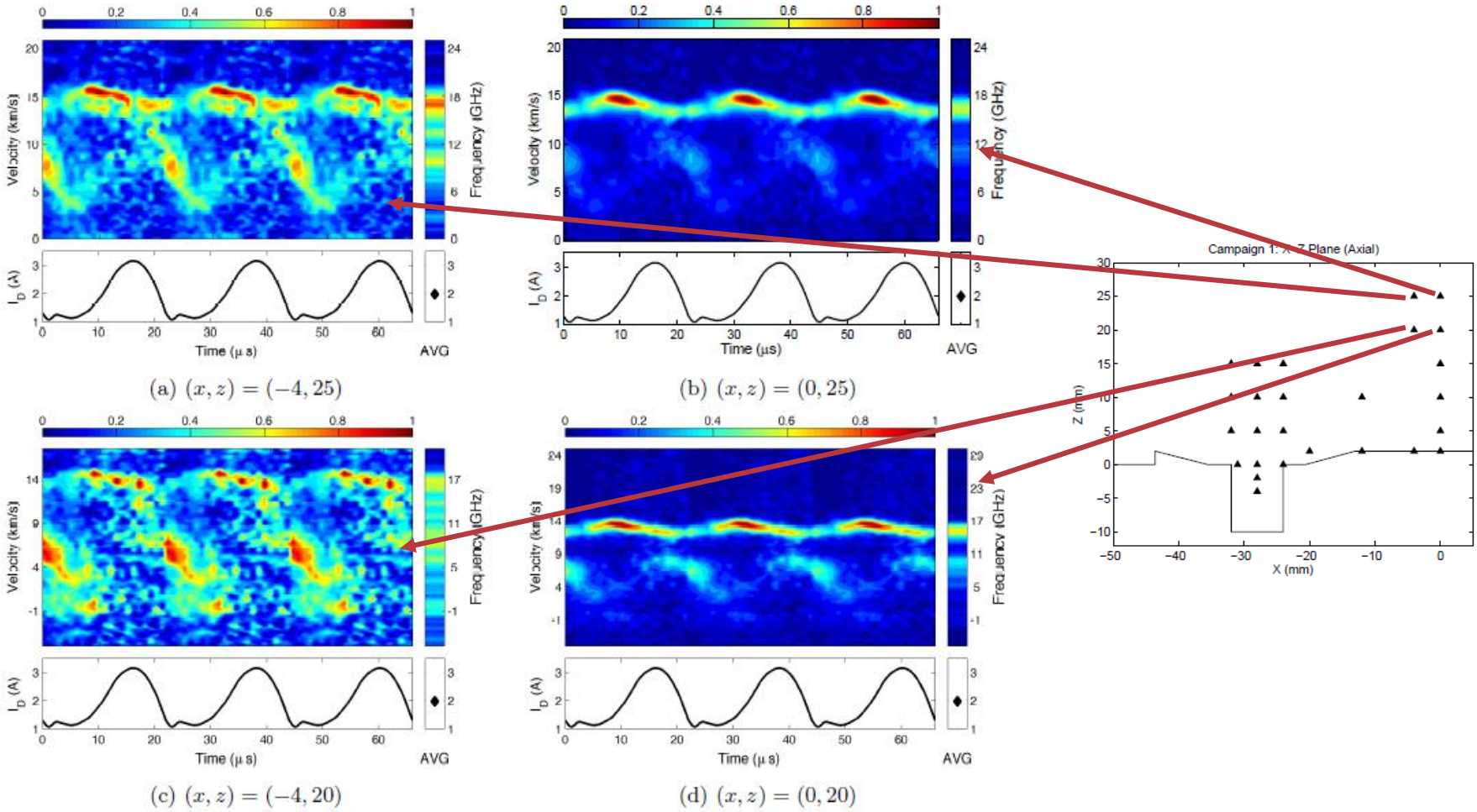
- Axial LIF traces show double populations near the thruster
- Suggests residual ionization is occurring in the central jet with local potential tied to breathing mode dynamics of channel



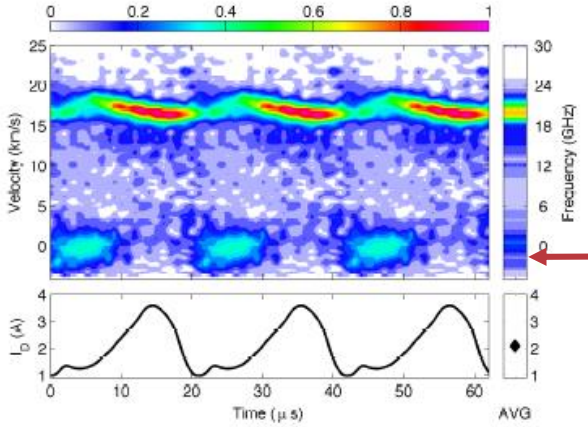
BHT-600 Results: Center Jet, Downstream

Slightly off-centerline

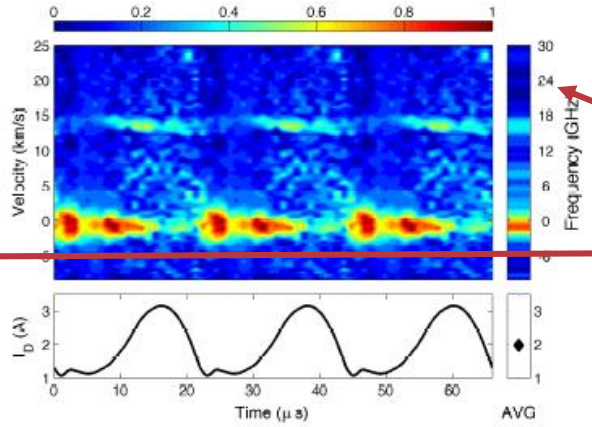
centerline



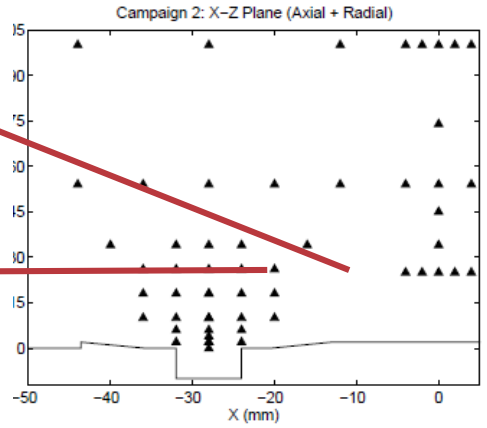
BHT-600 Results: Channel Outside



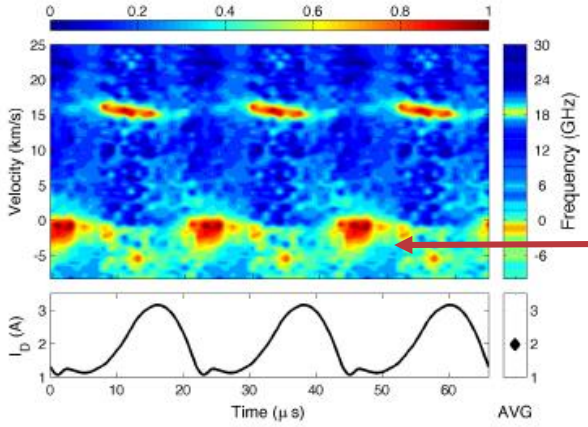
(a) $(x, z) = (-20, 10)$



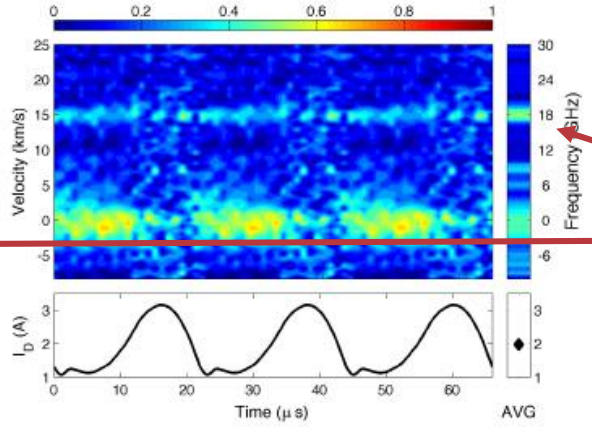
(b) $(x, z) = (-12, 10)$



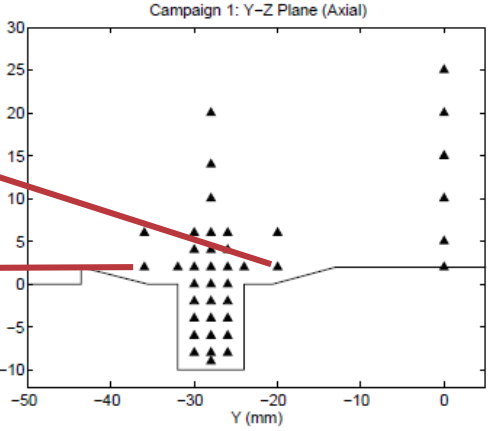
(c)



(c) $(y, z) = (-36, 2)$



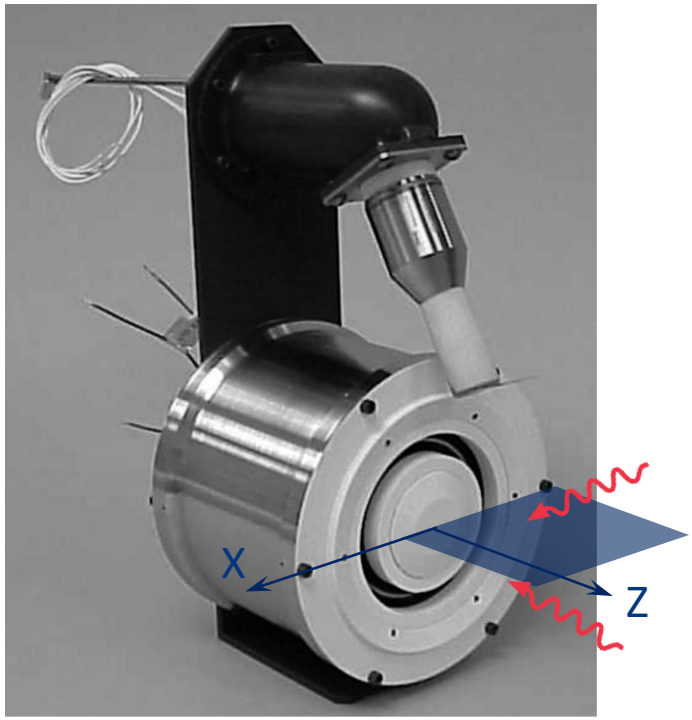
(d) $(y, z) = (-20, 2)$

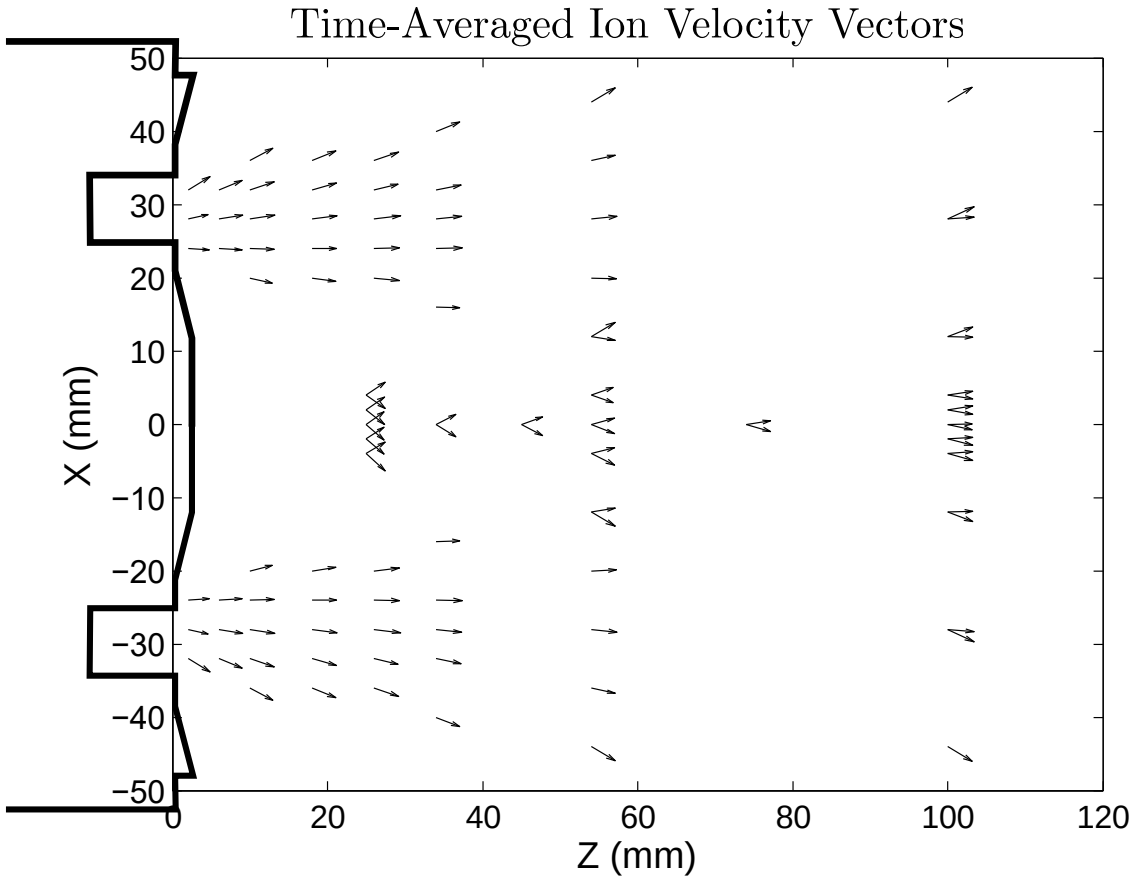


Thank You

BACKUP SLIDES

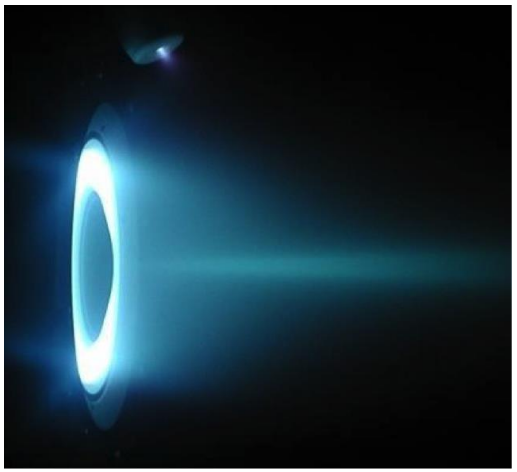
BHT-600 Results: 2-D Plume Ion Velocities

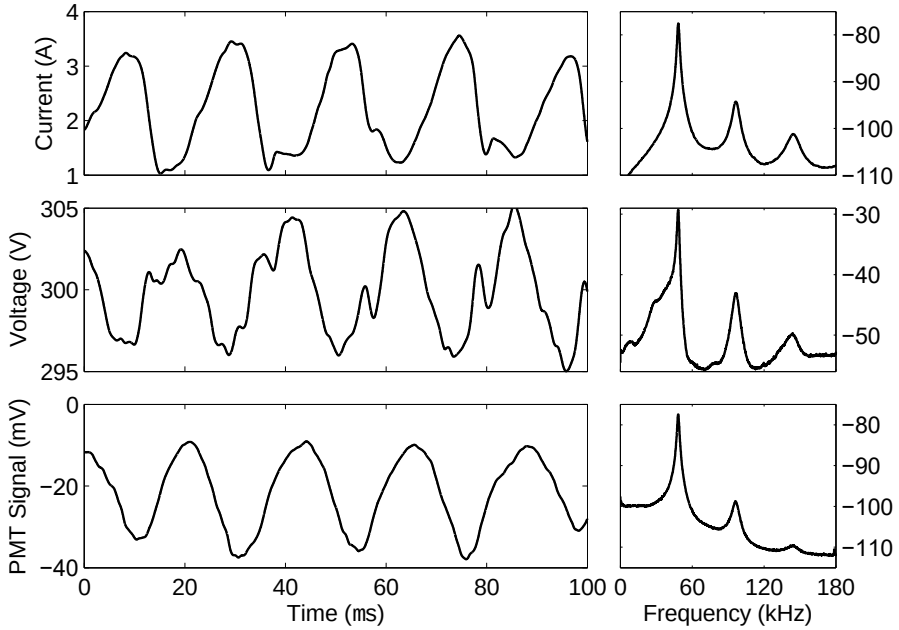




- Time-resolved axial + radial ion LIF in the plume

Hall Thrusters: Breathing Mode

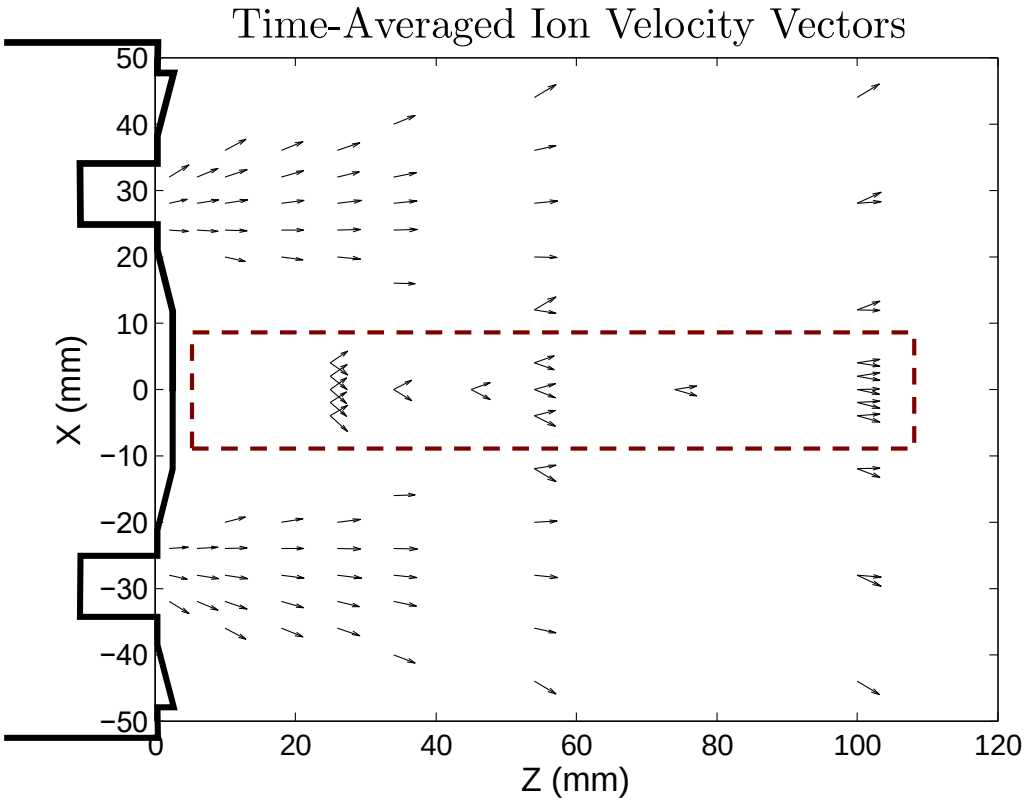


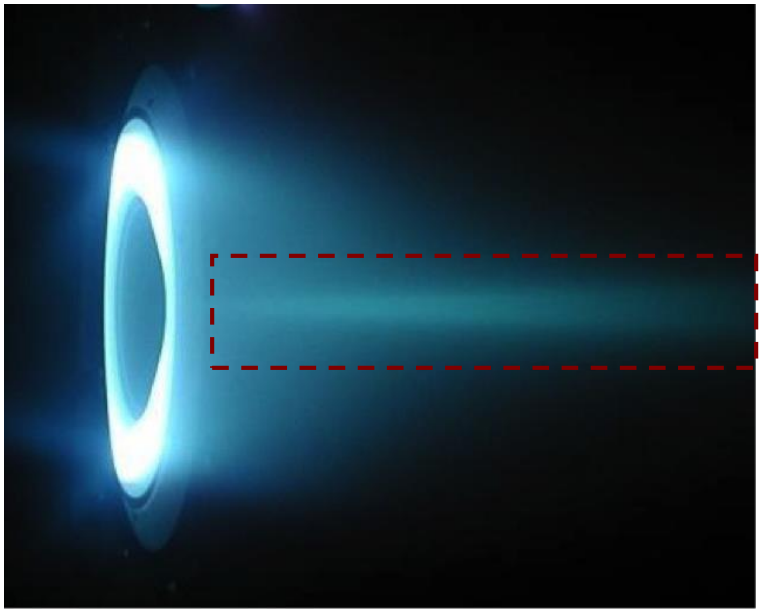


- Thruster works most efficiently when propellant is ionized in cycles
  - Neutrals fill channel
  - Ionization wave moves from exit to anode
  - Ions leave all at once
  - Wait for more neutrals to refill

Boeuf and Garrigues. *Journal of Applied Physics* **84**, 3541 (1998).
Bareilles et al. *Physics of Plasmas* **11**, 3035 (2004).

BHT-600 Results: Center Jet

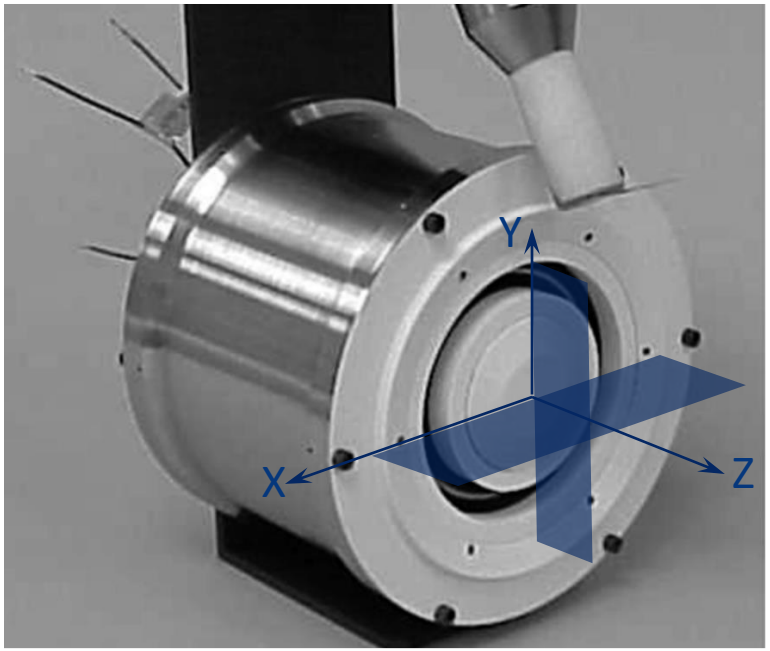


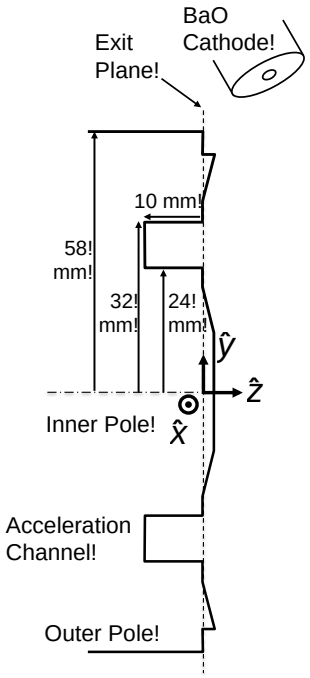


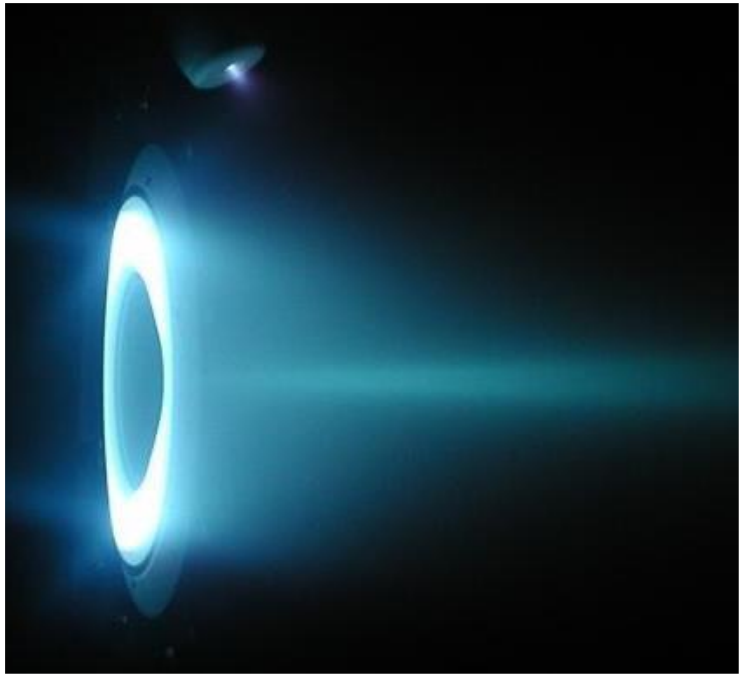
- Luminous central jet is not well understood and usually appears in efficient operating modes

V. Hruby et al. AIAA-1999-3534 (1999).

BHT-600 Hall Thruster







Anode Flow Rate: 2.21 mg/s Xe
Cathode Flow Rate: 147 μg/s Xe

Anode Voltage: 300 V
Anode Current: 2.05-2.15 A
Anode Power: 630 W

Channel ID: 24 mm
Channel OD: 32 mm
Channel Depth: 10 mm

Thrust: 39 mN
Specific Impulse: 1500 s
Anode Efficiency: 49 %

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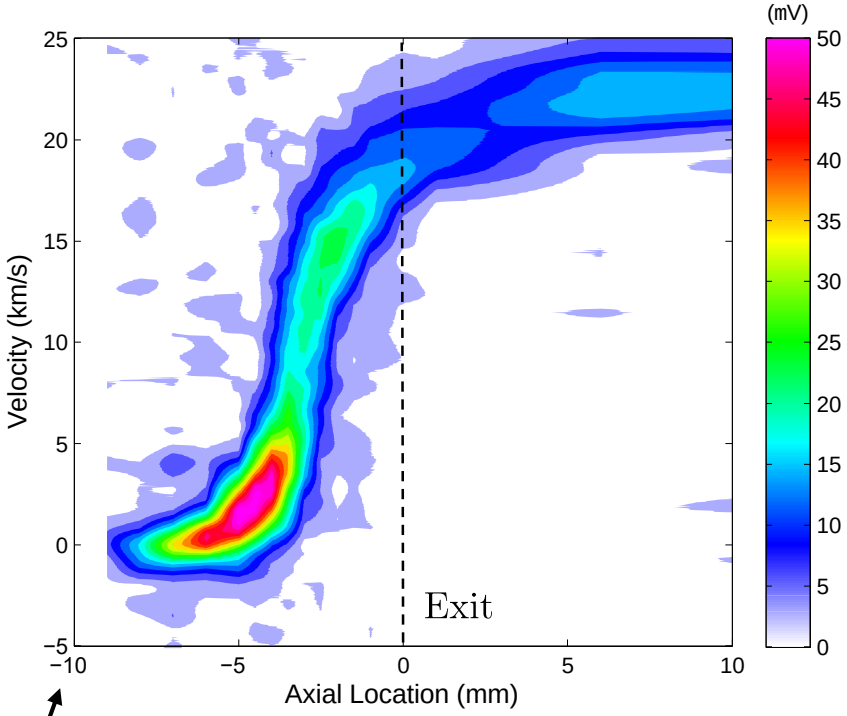
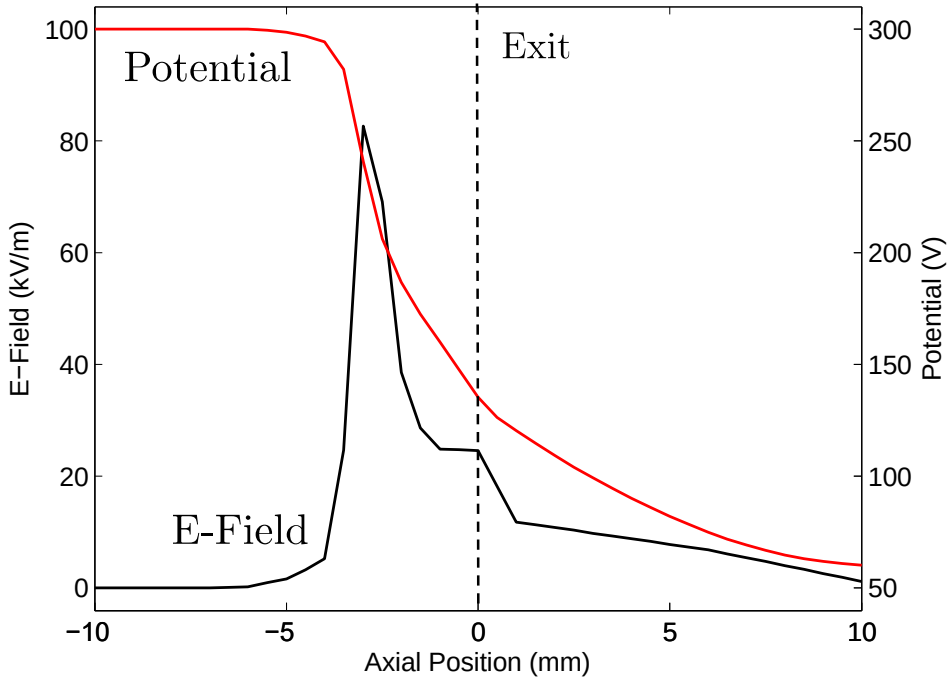
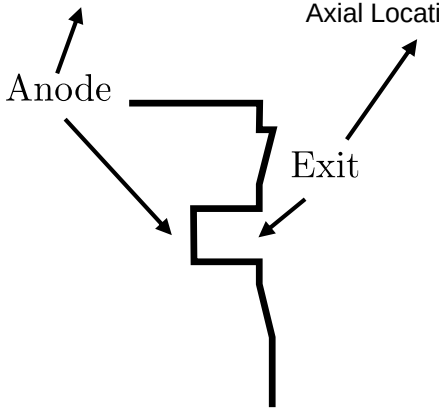
BHT-600 Results: Time-Averaged Channel





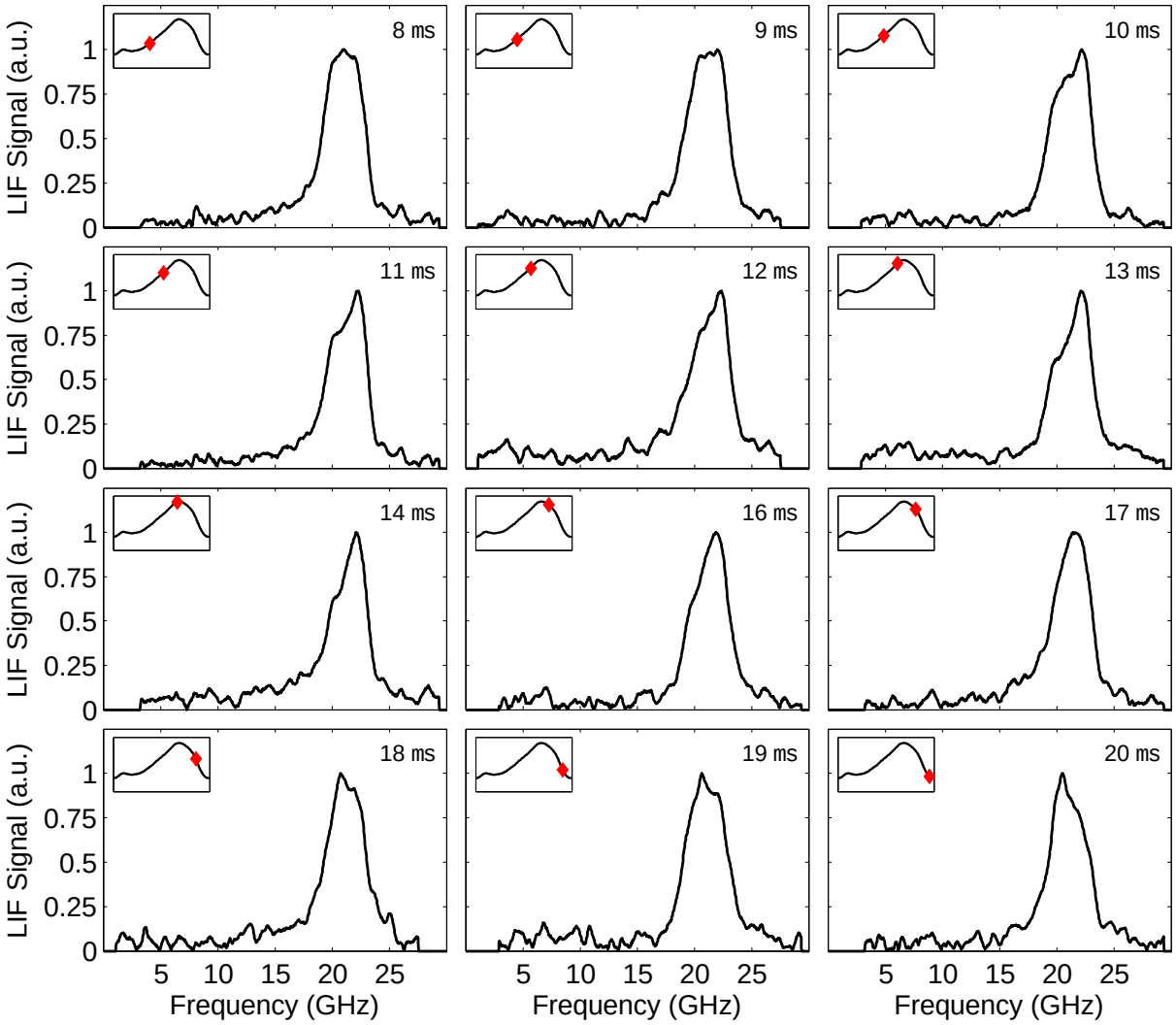


$$\text{Electric Field:} \quad E_z = \frac{m_i}{e}\langle v_z \rangle \frac{d\langle v_z \rangle}{dz}$$

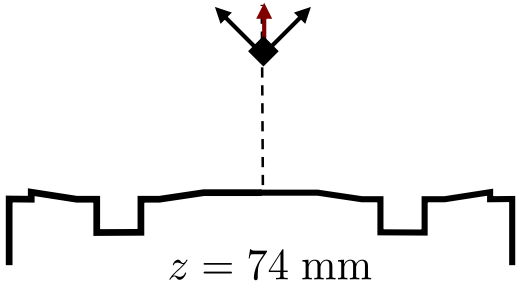
$$\text{Electric Potential:} \quad \phi(z) = \phi_{\mathrm{A}} - \int_{z_{\mathrm{A}}}^{z} E_z(\bar{z})\, d\bar{z}$$

- As usual, time-averaged data only tells part of the story…

BHT-600 Results: Center Jet, Time-Resolved

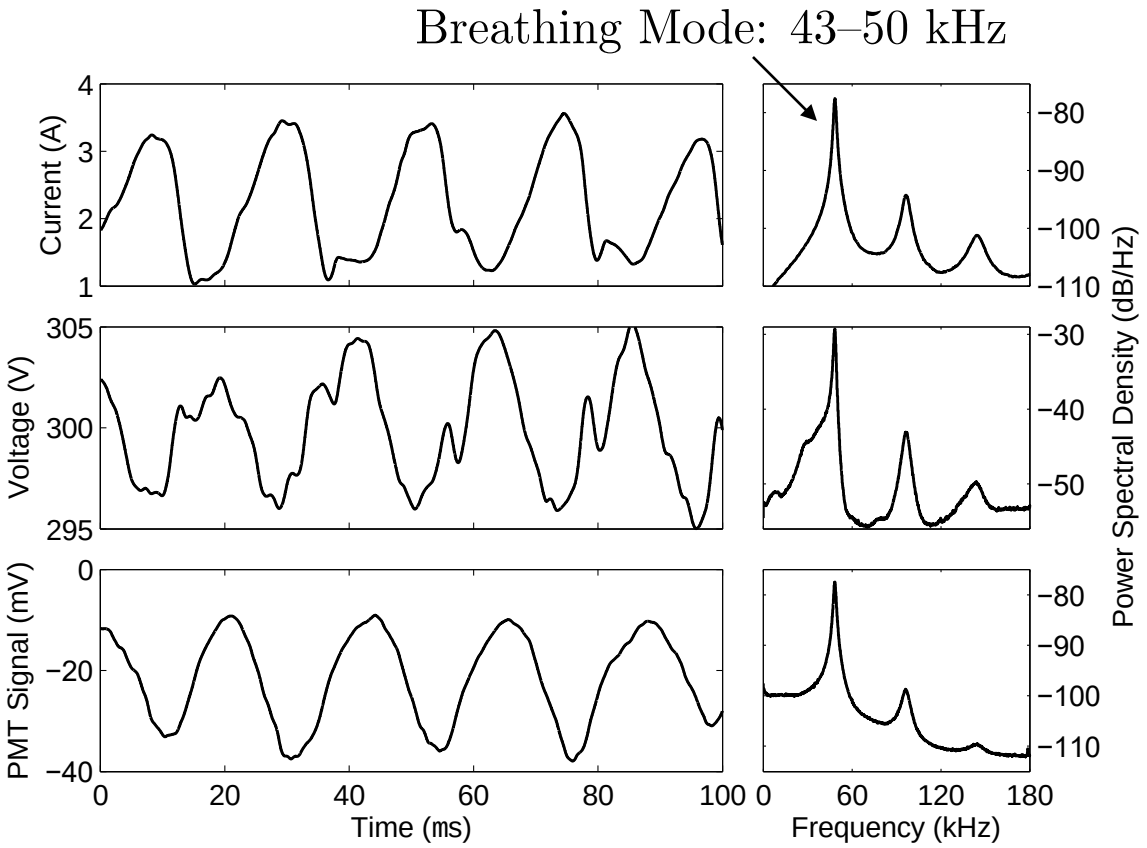


- Axial LIF traces show asymmetry in time:
  - Ion beam from one side of channel dominates for half cycle before switching
- Could be related to helical plasma wave seen previously in plume[1]

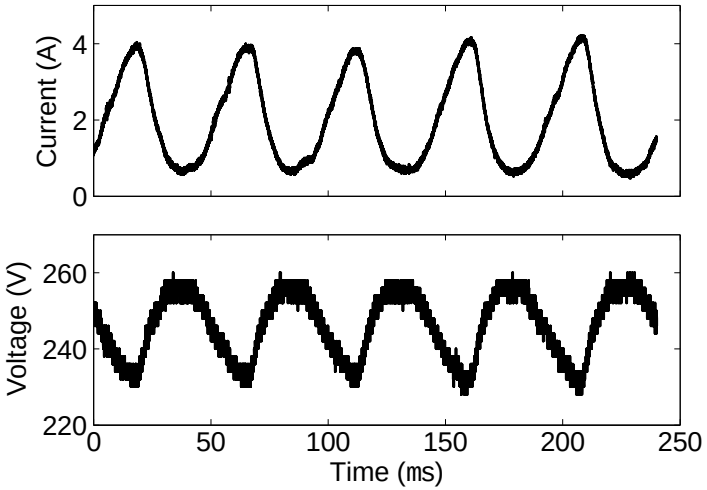


[1]A. W. Smith, PhD Thesis, Stanford University (2009)

BHT-600 Hall Thruster



BHT-600

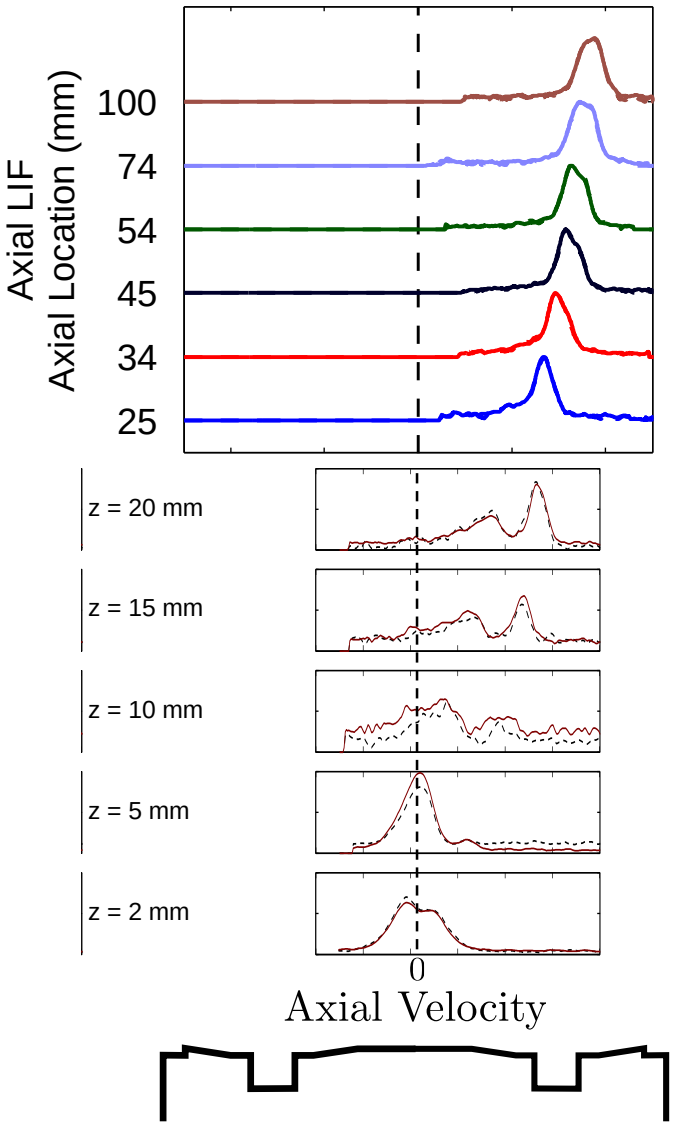


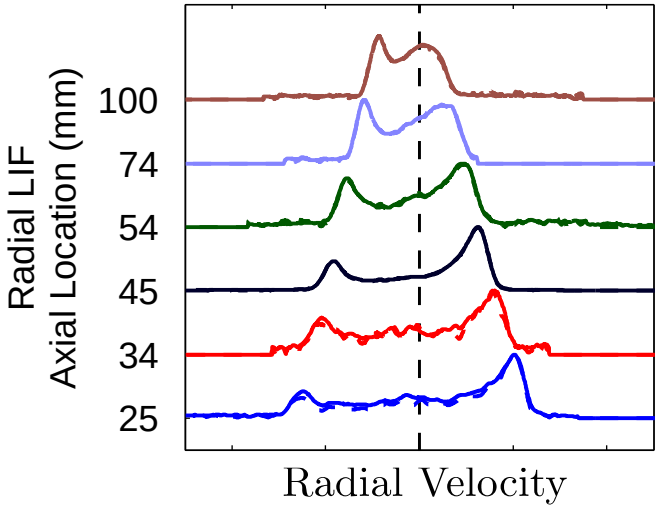
Z-70

- Current and voltage oscillations 180° out of phase in *both* thrusters
- Issue of power supply dynamics influencing thruster operation deserves more attention[1]

[1]W. Liqiu, et al. *Physics of Plasmas*, **18,** 063508 (2011).

BHT-600 Results: Center Jet, Time-Avg





- Double radial peaks = crossing beams
- Time-averaged radial ion LIF traces are asymmetric – small potential deviations
- Multiple axial ion populations observed closer to the thruster

BHT-600 Results: Particle Visualization